Old Past Papers - Integration

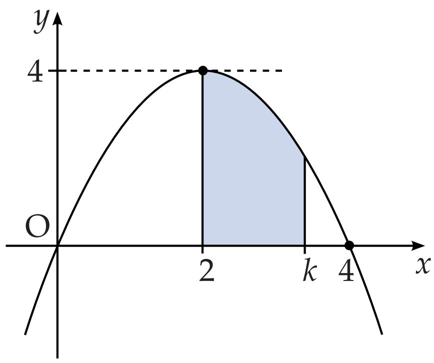
[SQA] 1. Find $\displaystyle\int \frac{(x^2-2)(x^2+2)}{x^2}\,dx,\ x \neq 0.$ **4**

Part	Marks	Level	Calc.	Content	Answer	U2 OC2
	4	C	CN	C14, C12, C13	$\frac{1}{3}x^3 + 4x^{-1} + c$	2001 P2 Q6

•[1] ss: start to write in standard form	•[1] $\dfrac{x^4-4}{x^2}$
•[2] pd: complete process	•[2] $x^2 - 4x^{-2}$
•[3] pd: integrate	•[3] $\frac{1}{3}x^3 + c$
•[4] pd: integrate a $-$ve index	•[4] $\dfrac{-4x^{-1}}{-1}$

[SQA] 2. The parabola shown crosses the x-axis at $(0,0)$ and $(4,0)$, and has a maximum at $(2,4)$.

The shaded area is bounded by the parabola, the x-axis and the lines $x=2$ and $x=k$.



(*a*) Find the equation of the parabola. **2**

(*b*) Hence show that the shaded area, A, is given by

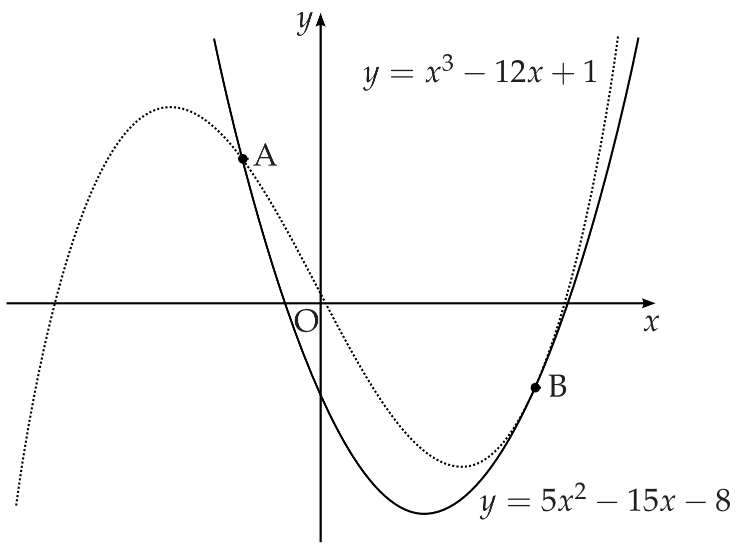
$A = -\frac{1}{3}k^3 + 2k^2 - \frac{16}{3}.$ **3**

Part	Marks	Level	Calc.	Content	Answer	U2 OC2
(*a*)	2	C	CN	A19	$y = 4x - x^2$	2000 P2 Q4
(*b*)	3	C	CN	C16	proof	

•[1] ic: state standard form	•[1] $ax(x-4)$
•[2] pd: process for x^2 coeff.	•[2] $a = -1$
•[3] ss: know to integrate	•[3] $\int_2^k$(function from (a))
•[4] pd: integrate correctly	•[4] $-\frac{1}{3}x^3 + 2x^2$
•[5] pd: process limits and complete proof	•[5] $-\frac{1}{3}k^3 + 2k^2 - \left(-\frac{8}{3} + 8\right)$

[SQA] 3. The diagram shows a sketch of the graphs of $y = 5x^2 - 15x - 8$ and $y = x^3 - 12x + 1$.

The two curves intersect at A and touch at B, i.e. at B the curves have a common tangent.



(*a*) (i) Find the x-coordinates of the point of the curves where the gradients are equal. **4**

(ii) By considering the corresponding y-coordinates, or otherwise, distinguish geometrically between the two cases found in part (i). **1**

(*b*) The point A is $(-1, 12)$ and B is $(3, -8)$.

Find the area enclosed between the two curves. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC2
(*ai*)	4	C	NC	C4	$x = \frac{1}{3}$ and $x = 3$	2000 P1 Q4
(*aii*)	1	C	NC	CGD	parallel and coincident	
(*b*)	5	C	NC	C17	$21\frac{1}{3}$	

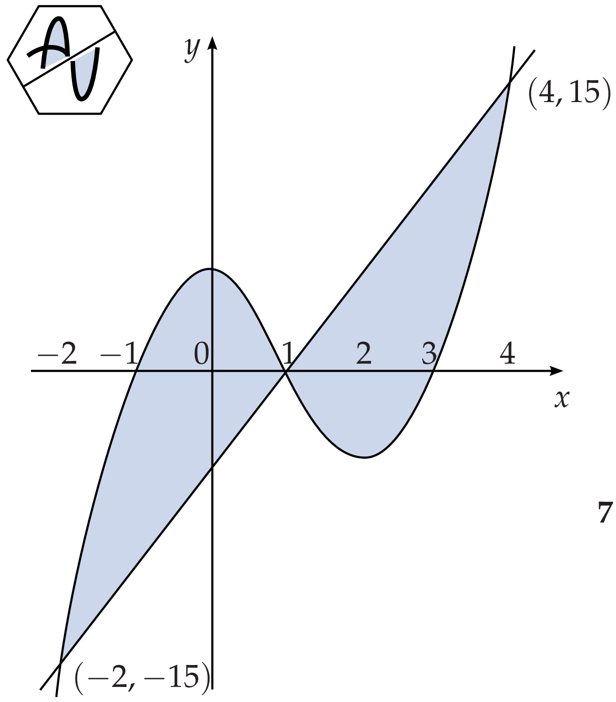
•1 ss: know to diff. and equate	•1 find derivatives and equate
•2 pd: differentiate	•2 $3x^2 - 12$ **and** $10x - 15$
•3 pd: form equation	•3 $3x^2 - 10x + 3 = 0$
•4 ic: interpret solution	•4 $x = 3, x = \frac{1}{3}$
•5 ic: interpret diagram	•5 tangents at $x = \frac{1}{3}$ are parallel, at $x = 3$ coincident
•6 ss: know how to find area between curves	•6 $\int$(cubic − parabola) **or** $\int$(cubic) − $\int$(parabola)
•7 ic: interpret limits	•7 $\int_{-1}^{3} \cdots dx$
•8 pd: form integral	•8 $\int (x^3 - 5x^2 + 3x + 9)dx$ or equiv.
•9 pd: process integration	•9 $\left[\frac{1}{4}x^4 - \frac{5}{3}x^3 + \frac{3}{2}x^2 + 9x\right]_{-1}^{3}$ or equiv.
•10 pd: process limits	•10 $21\frac{1}{3}$

[SQA] 4. A firm asked for a logo to be designed involving the letters A and U. Their initial sketch is shown in the hexagon.

A mathematical representation of the final logo is shown in the coordinate diagram.

The curve has equation $y = (x+1)(x-1)(x-3)$ and the straight line has equation $y = 5x - 5$. The point $(1,0)$ is the centre of half-turn symmetry.

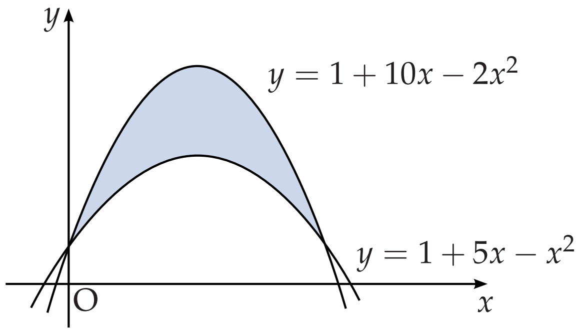
Calculate the total shaded area. **7**



Part	Marks	Level	Calc.	Content	Answer	U2 OC2
	7	C	CN	C17	$40\frac{1}{2}$ units^2	2001 P2 Q8

•[1] ss: express in standard form	•[1] $y = x^3 - 3x^2 - x + 3$
•[2] ss: split area and integrate	•[2] $\int_1^4 (\ldots)dx$ **or** $\int_{-2}^1 (\ldots)dx$
•[3] ss: subtract functions	•[3] $\int \left[(5x-5) - (x^3 - 3x^2 - x + 3)\right] dx$ **or** $\int \left[(x^3 - 3x^2 - x + 3) - (5x-5)\right] dx$
•[4] pd: process	•[4] $\int (-x^3 + 3x^2 + 6x - 8)dx$
•[5] pd: process	•[5] $\left[-\frac{1}{4}x^4 + x^3 + 3x^2 - 8x\right]$
•[6] pd: process	•[6] $20\frac{1}{4}$ **or** $-20\frac{1}{4}$ *depending on chosen integrals*
•[7] ic: use symmetry or otherwise for total area	•[7] $40\frac{1}{2}$

[SQA] 5. Calculate the shaded area enclosed between the parabolas with equations $y = 1 + 10x - 2x^2$ and $y = 1 + 5x - x^2$. **6**



Part	Marks	Level	Calc.	Content	Answer	U2 OC2
	6	C	CN	C17	$20\frac{5}{6}$	2002 P2 Q5

•1 ss: find intersections •2 ss: know to find limits •3 ss: know to integrate (upper − lower) •4 pd: simplify •5 pd: integrate •6 pd: process limits	•1 $1 + 10x - 2x^2 = 1 + 5x - x^2$ •2 $x = 0, 5$ **and** $\int_0^5 ()$ •3 $\int ((1 + 10x - 2x^2) - (1 + 5x - x^2))\, dx$ •4 $\int (5x - x^2)\, dx$ •5 $\frac{5}{2}x^2 - \frac{1}{3}x^3$ •6 $20\frac{5}{6}$

[SQA] 6. A point moves in a straight line such that its acceleration a is given by $a = 2(4 - t)^{\frac{1}{2}}$, $0 \le t \le 4$. If it starts at rest, find an expression for the velocity v where $a = \frac{dv}{dt}$. **4**

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	4	C	NC	C18, C22	$V = -\frac{4}{3}(4 - t)^{\frac{3}{2}} + \frac{32}{3}$	2002 P2 Q8

•1 ss: know to integrate acceleration •2 pd: integrate •3 ic: use initial conditions with const. of int. •4 pd: process solution	•1 $V = \int (2(4 - t)^{\frac{1}{2}})\, dt$ *stated or implied by* •2 •2 $2 \times \frac{1}{-\frac{3}{2}}(4 - t)^{\frac{3}{2}}$ •3 $0 = 2 \times \frac{1}{-\frac{3}{2}}(4 - 0)^{\frac{3}{2}} + c$ •4 $c = 10\frac{2}{3}$

[SQA] 7. The graph of $y = f(x)$ passes through the point $\left(\frac{\pi}{9}, 1\right)$.

If $f'(x) = \sin(3x)$ express y in terms of x. **4**

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	4	A/B	NC	C18, C23	$y = -\frac{1}{3}\cos(3x) + \frac{7}{6}$	2000 P1 Q8

•1 ss: know to integrate •2 pd: integrate •3 ic: interpret $(\frac{\pi}{9}, 1)$ •4 pd: process	•1 $y = \int \sin(3x)\,dx$ *stated or implied by* •2 •2 $-\frac{1}{3}\cos(3x)$ •3 $1 = -\frac{1}{3}\cos(\frac{3\pi}{9}) + c$ or equiv. •4 $c = \frac{7}{6}$

[SQA] 8. A curve for which $\dfrac{dy}{dx} = 3\sin(2x)$ passes through the point $\left(\frac{5\pi}{12}, \sqrt{3}\right)$.

Find y in terms of x. **4**

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	4	A/B	CN	C18, C23	$y = -\frac{3}{2}\cos(2x) + \frac{1}{4}\sqrt{3}$	2001 P2 Q10

•1 pd: integrate trig function •2 pd: integrate composite function •3 ss: use given point to find "c" •4 pd: evaluate "c"	•1 $\int 3\sin(2x)\,dx$ *stated or implied by* •2 •2 $-\frac{3}{2}\cos(2x)$ •3 $\sqrt{3} = -\frac{3}{2}\cos(2 \times \frac{5}{12}\pi) + c$ •4 $c = \frac{1}{4}\sqrt{3}$ $(\approx 0{\cdot}4)$

[SQA] 9. Find $\displaystyle\int \frac{1}{(7-3x)^2}\,dx$. **2**

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	2	A/B	CN	C22, C14	$\dfrac{1}{3(7-3x)} + c$	2000 P2 Q10

•1 pd: integrate function •2 pd: deal with function of function	•1 $\frac{1}{-1}(7-3x)^{-1}$ •2 $\times \frac{1}{-3}$

[END OF QUESTIONS]