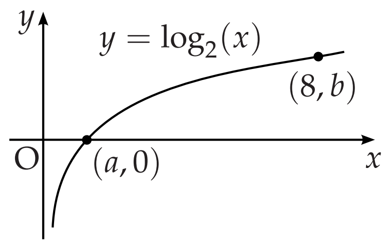
Old Past Papers - Functions and Graphs

[SQA] 1. The diagram shows a sketch of part of the graph of $y = \log_2(x)$.



(a) State the values of a and b. **1**

(b) Sketch the graph of $y = \log_2(x+1) - 3$. **3**

Part	Marks	Level	Calc.	Content	Answer	U1 OC2
(a)	1	A/B	CN	A7	$a = 1, b = 3$	2001 P1 Q10
(b)	3	A/B	CN	A3	sketch	

•1 pd: use $\log_p q = 0 \Rightarrow q = 1$ and evaluate $\log_p p^k$ •2 ss: use a translation •3 ic: identify one point •4 ic: identify a second point	•1 $a = 1$ **and** $b = 3$ •2 a "log-shaped" graph of the same orientation •3 sketch passes through $(0, -3)$ (labelled) •4 sketch passes through $(7, 0)$ (labelled)

[SQA] 2. $f(x) = 3 - x$ and $g(x) = \dfrac{3}{x}, x \neq 0$.

(a) Find $p(x)$ where $p(x) = f(g(x))$. **2**

(b) If $q(x) = \dfrac{3}{3-x}, x \neq 3$, find $p(q(x))$ in its simplest form. **3**

Part	Marks	Level	Calc.	Content	Answer	U1 OC2
(a)	2	C	CN	A4	$3 - \frac{3}{x}$	2000 P2 Q3
(b)	2	C	CN	A4	x	
(b)	1	A/B	CN	A4		

•1 ic: interpret composite func. •2 pd: process •3 ic: interpret composite func. •4 pd: process •5 pd: process	•1 $f\left(\frac{3}{x}\right)$ *stated or implied by* •2 •2 $3 - \frac{3}{x}$ •3 $p\left(\frac{3}{3-x}\right)$ *stated or implied by* •4 •4 $3 - \frac{3}{\frac{3}{3-x}}$ •5 x

[SQA] 3. Given $f(x) = x^2 + 2x - 8$, express $f(x)$ in the form $(x + a)^2 - b$. **2**

Part	Marks	Level	Calc.	Content	Answer	U1 OC2
	2	C	NC	A5	$(x+1)^2 - 9$	2001 P1 Q4

$\bullet^1$ ss: e.g. start to complete square $\bullet^2$ pd: complete process	$\bullet^1$ $(x+1)^2 \ldots$ $\bullet^2$ $(x+1)^2 - 9$ **or** $\bullet^1$ $a = 1$ $\bullet^2$ $b = 9$ **or** $\bullet^1$ $x^2 + 2x - 8 \equiv x^2 + 2ax + a^2 - b$ $\bullet^2$ $a = 1$ and $b = 9$

[SQA] 4. (*a*) Express $f(x) = x^2 - 4x + 5$ in the form $f(x) = (x - a)^2 + b$. **2**

(*b*) On the same diagram sketch:

(i) the graph of $y = f(x)$;

(ii) the graph of $y = 10 - f(x)$. **4**

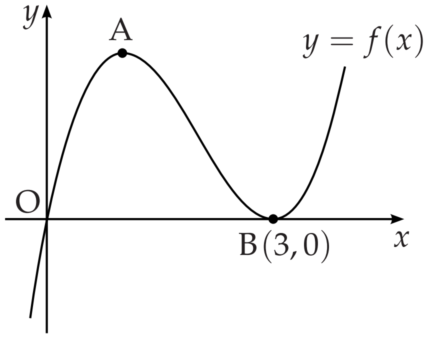
(*c*) Find the range of values of x for which $10 - f(x)$ is positive. **1**

Part	Marks	Level	Calc.	Content	Answer	U1 OC2
(*a*)	2	C	NC	A5	$a = 2, b = 1$	2002 P1 Q7
(*b*)	4	C	NC	A3	sketch	
(*c*)	1	C	NC	A16, A6	$-1 < x < 5$	

$\bullet^1$ pd: process, e.g. completing the square $\bullet^2$ pd: process, e.g. completing the square $\bullet^3$ ic: interpret minimum $\bullet^4$ ic: interpret y-intercept $\bullet^5$ ss: reflect in x-axis $\bullet^6$ ss: translate parallel to y-axis $\bullet^7$ ic: interpret graph	$\bullet^1$ $a = 2$ $\bullet^2$ $b = 1$ $\bullet^3$ *any two from:* parabola; min. t.p. $(2, 1)$; $(0, 5)$ $\bullet^4$ the remaining one from above list $\bullet^5$ reflecting in x-axis $\bullet^6$ translating $+10$ units, parallel to y-axis $\bullet^7$ $(-1, 5)$ i.e. $-1 < x < 5$

[SQA] 5. A sketch of the graph of $y = f(x)$ where $f(x) = x^3 - 6x^2 + 9x$ is shown below.

The graph has a maximum at A and a minimum at B$(3,0)$.



(*a*) Find the coordinates of the turning point at A. **4**

(*b*) Hence sketch the graph of $y = g(x)$ where $g(x) = f(x+2) + 4$.

Indicate the coordinates of the turning points. There is no need to calculate the coordinates of the points of intersection with the axes. **2**

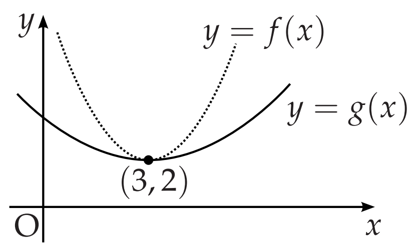
(*c*) Write down the range of values of k for which $g(x) = k$ has 3 real roots. **1**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
(*a*)	4	C	NC	C8	$A(1,4)$	2000 P1 Q2
(*b*)	2	C	NC	A3	sketch (translate 4 up, 2 left)	
(*c*)	1	A/B	NC	A2	$4 < k < 8$	

•1 ss: know to differentiate •2 pd: differentiate correctly •3 ss: know gradient = 0 •4 pd: process •5 ic: interpret transformation •6 ic: interpret transformation •7 ic: interpret sketch	•1 $\frac{dy}{dx} = \ldots$ •2 $\frac{dy}{dx} = 3x^2 - 12x + 9$ •3 $3x^2 - 12x + 9 = 0$ •4 $A = (1,4)$ translate $f(x)$ 4 units up, 2 units left •5 sketch with coord. of $A'(-1,8)$ •6 sketch with coord. of $B'(1,4)$ •7 $4 < k < 8$ (accept $4 \leq k \leq 8$)

[SQA] 6. The diagram shows the graphs of two quadratic functions $y = f(x)$ and $y = g(x)$. Both graphs have a minimum turning point at $(3,2)$.

Sketch the graph of $y = f'(x)$ and on the same diagram sketch the graph of $y = g'(x)$. **2**



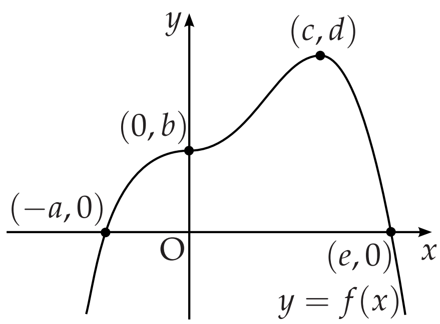
Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	2	C	CN	A3	sketch	2001 P1 Q9

•[1] ss: use $\frac{d}{dx}$(quadratic) $=$ linear •[2] ic: interpret stationary point	•[1] st. line for f' though $(3,0)$, $m_{f'} > 0$ •[2] st. line for g' through $(3,0)$, $m_{f'} > m_{g'} > 0$

[SQA] 7. The graph of a function f intersects the x-axis at $(-a,0)$ and $(e,0)$ as shown.

There is a point of inflexion at $(0,b)$ and a maximum turning point at (c,d).

Sketch the graph of the derived function f'. **3**



Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	3	C	CN	A3, C11	sketch	2002 P1 Q6

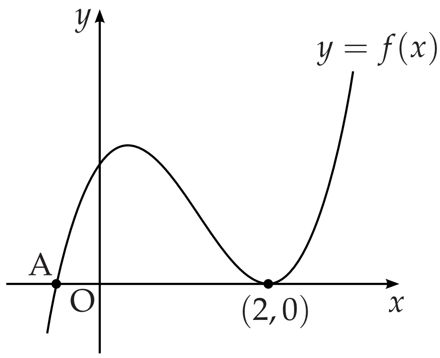
•[1] ic: interpret stationary points •[2] ic: interpret main body of f •[3] ic: interpret tails of f	•[1] roots at 0 and c (*accept a statement to this effect*) •[2] min. at LH root, max. between roots •[3] both 'tails' correct

[SQA] 8. The diagram shows part of the graph of the curve with equation $y = 2x^3 - 7x^2 + 4x + 4$.

(*a*) Find the x-coordinate of the maximum turning point. **5**

(*b*) Factorise $2x^3 - 7x^2 + 4x + 4$. **3**

(*c*) State the coordinates of the point A and hence find the values of x for which $2x^3 - 7x^2 + 4x + 4 < 0$. **2**



Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	5	C	NC	C8	$x = \frac{1}{3}$	2002 P2 Q3
(*b*)	3	C	NC	A21	$(x-2)(2x+1)(x-2)$	
(*c*)	2	C	NC	A6	$A(-\frac{1}{2}, 0)$, $x < -\frac{1}{2}$	

•1 ss: know to differentiate	•1 $f'(x) = \ldots$
•2 pd: differentiate	•2 $6x^2 - 14x + 4$
•3 ss: know to set derivative to zero	•3 $6x^2 - 14x + 4 = 0$
•4 pd: start solving process of equation	•4 $(3x-1)(x-2)$
•5 pd: complete solving process	•5 $x = \frac{1}{3}$
•6 ss: strategy for cubic, e.g. synth. division	•6 $\begin{array}{c\|cccc} \cdots & 2 & -7 & 4 & 4 \\ & & \cdots & \cdots & \cdots \\ \hline & \cdots & \cdots & \cdots & \boxed{0} \end{array}$
•7 ic: extract quadratic factor	•7 $2x^2 - 3x - 2$
•8 pd: complete the cubic factorisation	•8 $(x-2)(2x+1)(x-2)$
•9 ic: interpret the factors	•9 $A(-\frac{1}{2}, 0)$
•10 ic: interpret the diagram	•10 $x < -\frac{1}{2}$

[SQA] 9. Functions $f(x) = \sin x$, $g(x) = \cos x$ and $h(x) = x + \frac{\pi}{4}$ are defined on a suitable set of real numbers.

(*a*) Find expressions for:

(i) $f(h(x))$;

(ii) $g(h(x))$. **2**

(*b*) (i) Show that $f(h(x)) = \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x$.

(ii) Find a similar expression for $g(h(x))$ and hence solve the equation $f(h(x)) - g(h(x)) = 1$ for $0 \leq x \leq 2\pi$. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(*a*)	2	C	NC	A4	(i) $sin(x + \frac{\pi}{4})$, (ii) $cos(x + \frac{\pi}{4})$	2001 P1 Q7
(*b*)	5	C	NC	T8, T7	(i) proof, (ii) $x = \frac{\pi}{4}, \frac{3\pi}{4}$	

•1 ic: interpret composite functions	•1 $\sin(x + \frac{\pi}{4})$
•2 ic: interpret composite functions	•2 $\cos(x + \frac{\pi}{4})$
•3 ss: expand $\sin(x + \frac{\pi}{4})$	•3 $\sin x \cos \frac{\pi}{4} + \cos x \sin \frac{\pi}{4}$ **and complete**
•4 ic: interpret	•4 $g(h(x)) = \frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x$
•5 ic: substitute	•5 $(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x) - (\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x)$
•6 pd: start solving process	•6 $\frac{2}{\sqrt{2}}\sin x$
•7 pd: process	•7 $x = \frac{\pi}{4}, \frac{3\pi}{4}$ *accept only radians*

[SQA] 10. Functions f and g are defined on suitable domains by $f(x) = \sin(x^\circ)$ and $g(x) = 2x$.

(*a*) Find expressions for:

(i) $f(g(x))$;

(ii) $g(f(x))$. **2**

(*b*) Solve $2f(g(x)) = g(f(x))$ for $0 \leq x \leq 360$. **5**

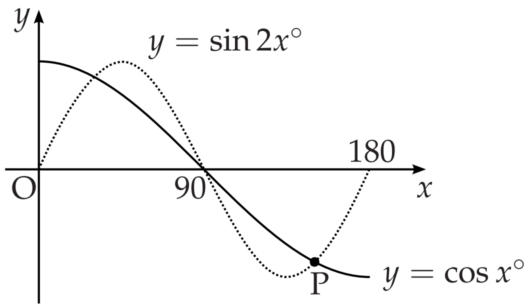
Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(*a*)	2	C	CN	A4	(i) $\sin(2x^\circ)$, (ii) $2\sin(x^\circ)$	2002 P1 Q3
(*b*)	5	C	CN	T10	$0^\circ, 60^\circ, 180^\circ, 300^\circ, 360^\circ$	

•1 ic: interpret $f(g(x))$	•1 $\sin(2x^\circ)$
•2 ic: interpret $g(f(x))$	•2 $2\sin(x^\circ)$
•3 ss: equate for intersection	•3 $2\sin(2x^\circ) = 2\sin(x^\circ)$
•4 ss: substitute for $\sin 2x$	•4 appearance of $2\sin(x^\circ)\cos(x^\circ)$
•5 pd: extract a common factor	•5 $2\sin(x^\circ)\,(2\cos(x^\circ) - 1)$
•6 pd: solve a 'common factor' equation	•6 $\sin(x^\circ) = 0$ **and** $0, 180, 360$
•7 pd: solve a 'linear' equation	•7 $\cos(x^\circ) = \frac{1}{2}$ **and** $60, 300$
	or
	•6 $\sin(x^\circ) = 0$ **and** $\cos(x^\circ) = \frac{1}{2}$
	•7 $0, 60, 180, 300, 360$

[SQA] 11. (*a*) Solve the equation $\sin 2x^\circ - \cos x^\circ = 0$ in the interval $0 \leq x \leq 180$. **4**

(*b*) The diagram shows parts of two trigonometric graphs, $y = \sin 2x^\circ$ and $y = \cos x^\circ$.

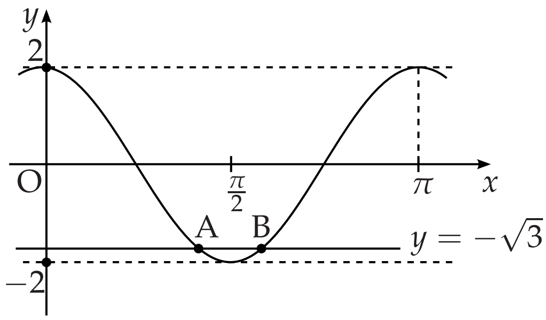
Use your solutions in (*a*) to write down the coordinates of the point P. **1**



Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(*a*)	4	C	NC	T10	$30, 90, 150$	2001 P1 Q5
(*b*)	1	C	NC	T3	$(150, -\frac{\sqrt{3}}{2})$	

•1 ss: use double angle formula	•1 $2\sin x^\circ \cos x^\circ$
•2 pd: factorise	•2 $\cos x^\circ(2\sin x^\circ - 1)$
•3 pd: process	•3 $\cos x^\circ = 0$, $\sin x^\circ = \frac{1}{2}$
•4 pd: process	•4 $90, 30, 150$
	or
	•3 $\sin x^\circ = \frac{1}{2}$ and $x = 30, 150$
	•4 $\cos x^\circ = 0$ and $x = 90$
•5 ic: interpret graph	•5 $\left(150, -\frac{\sqrt{3}}{2}\right)$

[SQA] 12. The diagram shows the graph of a cosine function from 0 to π.



(*a*) State the equation of the graph. **1**

(*b*) The line with equation $y = -\sqrt{3}$ intersects this graph at point A and B.

Find the coordinates of B. **3**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(*a*)	1	C	NC	T4	$y = 2\cos 2x$	2002 P1 Q8
(*b*)	3	C	NC	T7	$B(\frac{7\pi}{12}, -\sqrt{3})$	

•[1] ic: interpret graph	•[1] $2\cos 2x$
•[2] ss: equate equal parts •[3] pd: solve linear trig equation in radians •[4] ic: interpret result	•[1] $2\cos 2x = -\sqrt{3}$ •[2] $2x = \frac{5\pi}{6}, \frac{7\pi}{6}$ •[3] $x = \frac{7\pi}{12}$

[END OF QUESTIONS]