Old Past Papers - Differentiation

[SQA] 1. Find the coordinates of the point on the curve $y = 2x^2 - 7x + 10$ where the tangent to the curve makes an angle of 45° with the positive direction of the x-axis. **4**

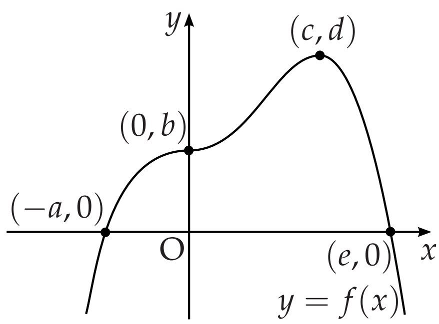
Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	4	C	NC	G2, C4	$(2, 4)$	2002 P1 Q4

•1 sp: know to diff., and differentiate	•1 $\frac{dy}{dx} = 4x - 7$
•2 pd: process gradient from angle	•2 $m_{\text{tang}} = \tan 45^\circ = 1$
•3 ss: equate equivalent expressions	•3 $4x - 7 = 1$
•4 pd: solve and complete	•4 $(2, 4)$

[SQA] 2. The graph of a function f intersects the x-axis at $(-a, 0)$ and $(e, 0)$ as shown.

There is a point of inflexion at $(0, b)$ and a maximum turning point at (c, d).

Sketch the graph of the derived function f'. **3**



Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	3	C	CN	A3, C11	sketch	2002 P1 Q6

•1 ic: interpret stationary points	•1 roots at 0 and c (*accept a statement to this effect*)
•2 ic: interpret main body of f	•2 min. at LH root, max. between roots
•3 ic: interpret tails of f	•3 both 'tails' correct

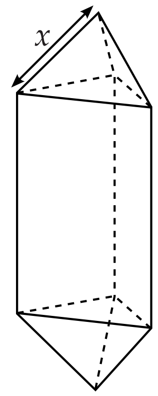
[SQA] 3. A goldsmith has built up a solid which consists of a triangular prism of fixed volume with a regular tetrahedron at each end.

The surface area, A, of the solid is given by

$$A(x) = \frac{3\sqrt{3}}{2}\left(x^2 + \frac{16}{x}\right)$$

where x is the length of each edge of the tetrahedron.

Find the value of x which the goldsmith should use to minimise the amount of gold plating required to cover the solid.



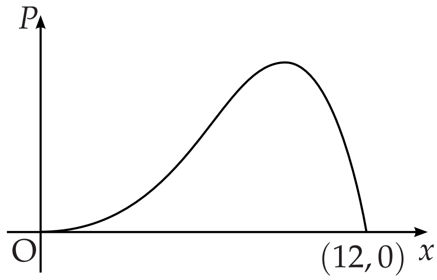
6

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	6	A/B	CN	C11	$x = 2$	2000 P2 Q6

•[1] ss: know to differentiate •[2] pd: process •[3] ss: know to set $f'(x) = 0$ •[4] pd: deal with x^{-2} •[5] pd: process •[6] ic: check for minimum	•[1] $A'(x) = \ldots$ •[2] $\frac{3\sqrt{3}}{2}(2x - 16x^{-2})$ **or** $3\sqrt{3}x - 24\sqrt{3}x^{-2}$ •[3] $A'(x) = 0$ •[4] $-\frac{16}{x^2}$ **or** $-\frac{24\sqrt{3}}{x^2}$ •[5] $x = 2$ •[6] $\begin{array}{c\|ccc} x & 2^- & 2 & 2^+ \\ \hline A'(x) & -ve & 0 & +ve \end{array}$ so $x = 2$ is min.

[SQA] 4. A company spends x thousand pounds a year on advertising and this results in a profit of P thousand pounds. A mathematical model , illustrated in the diagram, suggests that P and x are related by $P = 12x^3 - x^4$ for $0 \leq x \leq 12$.

Find the value of x which gives the maximum profit.

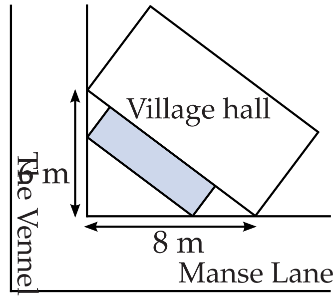


5

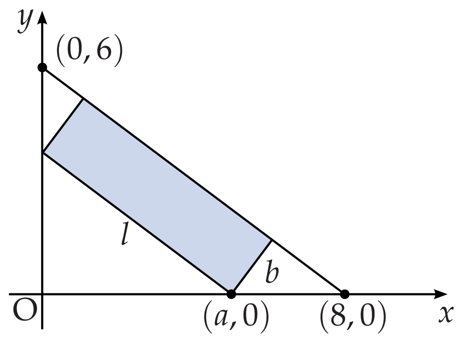
Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	5	C	NC	C11	$x = 9$	2001 P1 Q6

•[1] ss: start diff. process •[2] pd: process •[3] ss: set derivative to zero •[4] pd: process •[5] ic: interpret solutions	•[1] $\frac{dP}{dx} = 36x^2 \ldots$ **or** $\frac{dP}{dx} = \ldots - 4x^3$ •[2] $\frac{dP}{dx} = 36x^2 - 4x^3$ •[3] $\frac{dP}{dx} = 0$ •[4] $x = 0$ **and** $x = 9$ •[5] nature table about $x = 0$ **and** $x = 9$

[SQA] 5. The shaded rectangle on this map represents the planned extension to the village hall. It is hoped to provide the largest possible area for the extension.



The coordinate diagram represents the right angled triangle of ground behind the hall. The extension has length l metres and breadth b metres, as shown. One corner of the extension is at the point $(a, 0)$.



(*a*) (i) Show that $l = \frac{5}{4}a$.

(ii) Express b in terms of a and hence deduce that the area, A m^2, of the extension is given by $A = \frac{3}{4}a(8-a)$. **3**

(*b*) Find the value of a which produces the largest area of the extension. **4**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
(*a*)	3	A/B	CN	CGD	proof	2002 P2 Q10
(*b*)	4	A/B	CN	C11	$a = 4$	

•[1] ss: select strategy and carry through	•[1] proof of $l = \frac{5}{4}a$
•[2] ss: select strategy and carry through	•[2] $b = \frac{3}{5}(8-a)$
•[3] ic: complete proof	•[3] complete proof leading to $A = \ldots$
•[4] ss: know to set derivative to zero	•[4] $\frac{dA}{da} = \ldots = 0$
•[5] pd: differentiate	•[5] $6 - \frac{3}{2}a$
•[6] pd: solve equation	•[6] $a = 4$
•[7] ic: justify maximum, e.g. nature table	•[7] e.g. nature table, comp. the square

[SQA] 6. A curve has equation $y = x - \dfrac{16}{\sqrt{x}}$, $x > 0$.

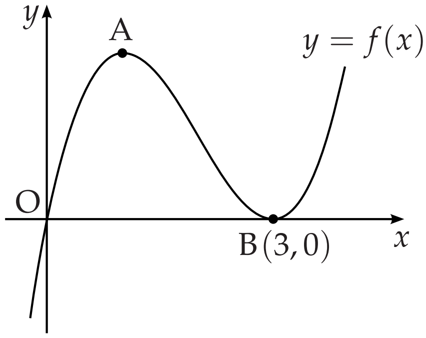
Find the equation of the tangent at the point where $x = 4$. **6**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	6	C	CN	C4, C5	$y = 2x - 12$	2001 P2 Q2

•[1] ic: find corresponding y-coord.	•[1] $(4, -4)$ *stated or implied by* •[6]
•[2] ss: express in standard form	•[2] $-16x^{-\frac{1}{2}}$
•[3] ss: start to differentiate	•[3] $\frac{dy}{dx} = 1\ldots$
•[4] pd: diff. fractional negative power	•[4] $\ldots + 8x^{-\frac{3}{2}}$
•[5] ss: find gradient of tangent	•[5] $m_{x=4} = 2$
•[6] ic: write down equ. of tangent	•[6] $y - (-4) = 2(x - 4)$

[SQA] 7. A sketch of the graph of $y = f(x)$ where $f(x) = x^3 - 6x^2 + 9x$ is shown below.

The graph has a maximum at A and a minimum at B$(3,0)$.



(*a*) Find the coordinates of the turning point at A. **4**

(*b*) Hence sketch the graph of $y = g(x)$ where $g(x) = f(x+2) + 4$.

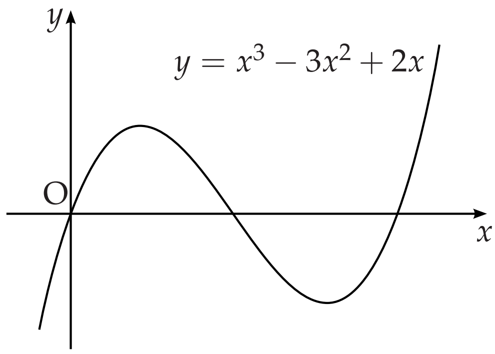
Indicate the coordinates of the turning points. There is no need to calculate the coordinates of the points of intersection with the axes. **2**

(*c*) Write down the range of values of k for which $g(x) = k$ has 3 real roots. **1**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
(*a*)	4	C	NC	C8	$A(1,4)$	2000 P1 Q2
(*b*)	2	C	NC	A3	sketch (translate 4 up, 2 left)	
(*c*)	1	A/B	NC	A2	$4 < k < 8$	

•[1] ss: know to differentiate	•[1] $\frac{dy}{dx} = \ldots$
•[2] pd: differentiate correctly	•[2] $\frac{dy}{dx} = 3x^2 - 12x + 9$
•[3] ss: know gradient $= 0$	•[3] $3x^2 - 12x + 9 = 0$
•[4] pd: process	•[4] $A = (1,4)$
	translate $f(x)$ 4 units up, 2 units left
•[5] ic: interpret transformation	•[5] sketch with coord. of $A'(-1,8)$
•[6] ic: interpret transformation	•[6] sketch with coord. of $B'(1,4)$
•[7] ic: interpret sketch	•[7] $4 < k < 8$ (accept $4 \le k \le 8$)

[SQA] 8. The diagram shows a sketch of the graph of $y = x^3 - 3x^2 + 2x$.



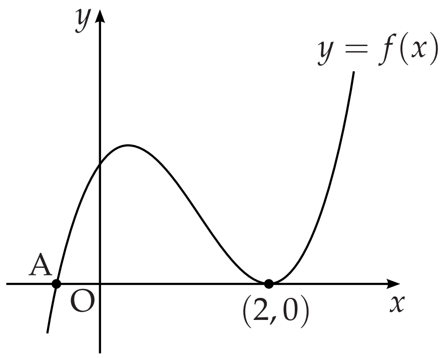
(*a*) Find the equation of the tangent to this curve at the point where $x = 1$. **5**

(*b*) The tangent at the point $(2, 0)$ has equation $y = 2x - 4$. Find the coordinates of the point where this tangent meets the curve again. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	5	C	CN	C5	$x + y = 1$	2000 P2 Q1
(*b*)	5	C	CN	A23, A22, A21	$(-1, -6)$	

•1 ss: know to differentiate	•1 $y' = \ldots$
•2 pd: differentiate correctly	•2 $3x^2 - 6x + 2$
•3 ss: know that gradient $= f'(1)$	•3 $y'(1) = -1$
•4 ss: know that y-coord $= f(1)$	•4 $y(1) = 0$
•5 ic: state equ. of line	•5 $y - 0 = -1(x - 1)$
•6 ss: equate equations	•6 $2x - 4 = x^3 - 3x^2 + 2x$
•7 pd: arrange in standard form	•7 $x^3 - 3x^2 + 4 = 0$
•8 ss: know how to solve cubic	•8 $\begin{array}{c\|cccc} \cdots & 1 & -3 & 0 & 4 \\ & & \cdots & \cdots & \cdots \\ \hline & \cdots & \cdots & \cdots & \cdots \end{array}$
•9 pd: process	•9 identify $x = -1$ from working
•10 ic: interpret	•10 $(-1, -6)$

[SQA] 9. The diagram shows part of the graph of the curve with equation $y = 2x^3 - 7x^2 + 4x + 4$.



(*a*) Find the x-coordinate of the maximum turning point. **5**

(*b*) Factorise $2x^3 - 7x^2 + 4x + 4$. **3**

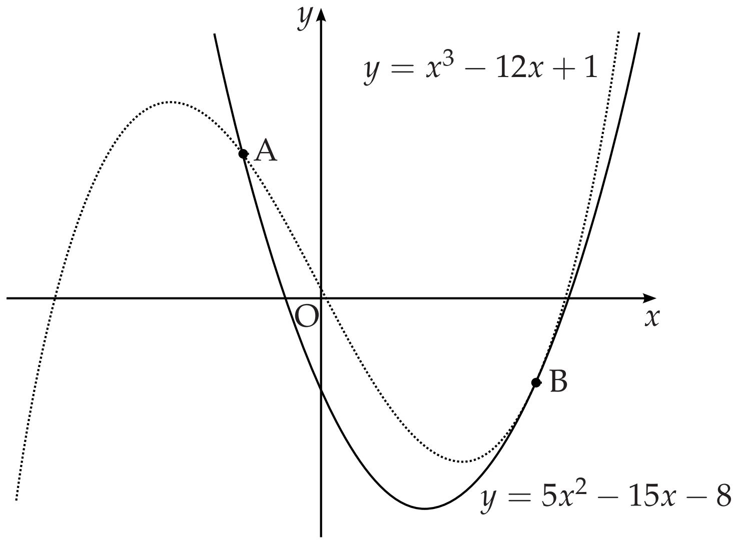
(*c*) State the coordinates of the point A and hence find the values of x for which $2x^3 - 7x^2 + 4x + 4 < 0$. **2**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	5	C	NC	C8	$x = \frac{1}{3}$	2002 P2 Q3
(*b*)	3	C	NC	A21	$(x-2)(2x+1)(x-2)$	
(*c*)	2	C	NC	A6	$A(-\frac{1}{2}, 0)$, $x < -\frac{1}{2}$	

•1 ss: know to differentiate	•1 $f'(x) = \ldots$
•2 pd: differentiate	•2 $6x^2 - 14x + 4$
•3 ss: know to set derivative to zero	•3 $6x^2 - 14x + 4 = 0$
•4 pd: start solving process of equation	•4 $(3x-1)(x-2)$
•5 pd: complete solving process	•5 $x = \frac{1}{3}$
•6 ss: strategy for cubic, e.g. synth. division	•6 $\begin{array}{c\|cccc} \cdots & 2 & -7 & 4 & 4 \\ & & \cdots & \cdots & \cdots \\ \hline & \cdots & \cdots & \cdots & \boxed{0} \end{array}$
•7 ic: extract quadratic factor	•7 $2x^2 - 3x - 2$
•8 pd: complete the cubic factorisation	•8 $(x-2)(2x+1)(x-2)$
•9 ic: interpret the factors	•9 $A(-\frac{1}{2}, 0)$
•10 ic: interpret the diagram	•10 $x < -\frac{1}{2}$

[SQA] 10. The diagram shows a sketch of the graphs of $y = 5x^2 - 15x - 8$ and $y = x^3 - 12x + 1$.

The two curves intersect at A and touch at B, i.e. at B the curves have a common tangent.



(*a*) (i) Find the x-coordinates of the point of the curves where the gradients are equal. **4**

(ii) By considering the corresponding y-coordinates, or otherwise, distinguish geometrically between the two cases found in part (i). **1**

(*b*) The point A is $(-1, 12)$ and B is $(3, -8)$.

Find the area enclosed between the two curves. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC2
(*ai*)	4	C	NC	C4	$x = \frac{1}{3}$ and $x = 3$	2000 P1 Q4
(*aii*)	1	C	NC	CGD	parallel and coincident	
(*b*)	5	C	NC	C17	$21\frac{1}{3}$	

•1 ss: know to diff. and equate	•1 find derivatives and equate
•2 pd: differentiate	•2 $3x^2 - 12$ **and** $10x - 15$
•3 pd: form equation	•3 $3x^2 - 10x + 3 = 0$
•4 ic: interpret solution	•4 $x = 3,\ x = \frac{1}{3}$
•5 ic: interpret diagram	•5 tangents at $x = \frac{1}{3}$ are parallel, at $x = 3$ coincident
•6 ss: know how to find area between curves	•6 $\int(\text{cubic} - \text{parabola})$ **or** $\int(\text{cubic}) - \int(\text{parabola})$
•7 ic: interpret limits	•7 $\int_{-1}^{3} \cdots dx$
•8 pd: form integral	•8 $\int(x^3 - 5x^2 + 3x + 9)dx$ or equiv.
•9 pd: process integration	•9 $\left[\frac{1}{4}x^4 - \frac{5}{3}x^3 + \frac{3}{2}x^2 + 9x\right]_{-1}^{3}$ or equiv.
•10 pd: process limits	•10 $21\frac{1}{3}$

[SQA] 11. Find the equation of the tangent to the curve $y = 2\sin(x - \frac{\pi}{6})$ at the point where $x = \frac{\pi}{3}$. **4**

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	4	C	CN	C5, C20	$y = \sqrt{3}x + 1 - \frac{\pi}{\sqrt{3}}$	2002 P2 Q6

•1 pd: find derivative	•1 $\frac{dy}{dx} = 2\cos(x - \frac{\pi}{6})$
•2 ss: know derivative at $x = \ldots$ represents grad.	•2 $m = \sqrt{3}$
•3 pd: find corresponding y-coordinate	•3 $y_{x=\frac{\pi}{3}} = 1$
•4 ic: state equation of tangent	•4 $y - 1 = \sqrt{3}(x - \frac{\pi}{3})$

[END OF QUESTIONS]