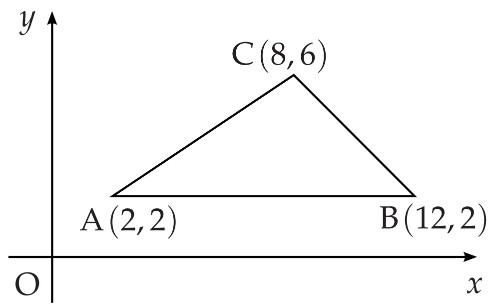
Old Past Papers - Circles

[SQA] 1. Triangle ABC has vertices A(2, 2), B(12, 2) and C(8, 6).



(a) Write down the equation of l_1, the perpendicular bisector of AB. **1**

(b) Find the equation of l_2, the perpendicular bisector of AC. **4**

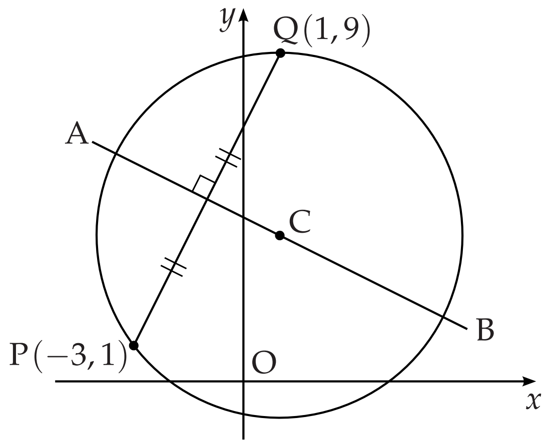
(c) Find the point of intersection of lines l_1 and l_2. **1**

(d) Hence find the equation of the circle passing through A, B and C. **2**

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
(a)	1	C	CN	G3, G7	$x = 7$	2001 P2 Q7
(b)	4	C	CN	G7	$3x + 2y = 23$	
(c)	1	C	CN	G8	$(7, 1)$	
(d)	2	A/B	CN	G8, G9, G10	$(x-7)^2 + (y-1)^2 = 26$	

•1 ic: state equation of a vertical line	•1 $x = 7$
•2 pd: process coord. of a midpoint	•2 midpoint $= (5, 4)$
•3 ss: find gradient of AC	•3 $m_{AC} = \frac{2}{3}$
•4 ic: state gradient of perpendicular	•4 $m_{\perp} = -\frac{3}{2}$
•5 ic: state equation of straight line	•5 $y - 4 = -\frac{3}{2}(x-5)$
•6 pd: find pt of intersection	•6 $x = 7, y = 1$
•7 ss: use standard form of circle equ.	•7 $(x-7)^2 + (y-1)^2$
•8 ic: find radius and complete	•8 $(x-7)^2 + (y-1)^2 = 26$
	or
	•7 $x^2 + y^2 - 14x - 2y + c = 0$
	•8 $c = 24$

[SQA] 2. (*a*) Find the equation of AB, the perpendicular bisector of the line joing the points $P(-3,1)$ and $Q(1,9)$. **4**



(*b*) C is the centre of a circle passing through P and Q. Given that QC is parallel to the y-axis, determine the equation of the circle. **3**

(*c*) The tangents at P and Q intersect at T.

Write down

(i) the equation of the tangent at Q

(ii) the coordinates of T. **2**

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
(*a*)	4	C	CN	G7	$x+2y=9$	2000 P2 Q2
(*b*)	3	C	CN	G10	$(x-1)^2+(y-4)^2=25$	
(*c*)	2	C	CN	G11, G8	(i) $y=9$, (ii) $T(-9,9)$	

$\bullet^1$ ss: know to use midpoint	$\bullet^1$ midpoint $=(-1,5)$
$\bullet^2$ pd: process gradient of PQ	$\bullet^2$ $m_{PQ}=\frac{9-1}{1-(-1)}$
$\bullet^3$ ss: know how to find perp. gradient	$\bullet^3$ $m_\perp=-\frac{1}{2}$
$\bullet^4$ ic: state equ. of line	$\bullet^4$ $y-5=-\frac{1}{2}(x-(-1))$
$\bullet^5$ ic: interpret "parallel to y-axis"	$\bullet^5$ $y_C=4$ *stated or implied by* $\bullet^7$
$\bullet^6$ pd: process radius	$\bullet^6$ radius $=5$ or equiv. *stated or implied by* $\bullet^7$
$\bullet^7$ ic: state equ. of circle	$\bullet^7$ $(x-1)^2+(y-4)^2=25$
$\bullet^8$ ic: interpret diagram	$\bullet^8$ $y=9$
$\bullet^9$ ss: know to use equ. of AB	$\bullet^9$ T$=(-9,9)$

[SQA] 3. Circle P has equation $x^2 + y^2 - 8x - 10y + 9 = 0$. Circle Q has centre $(-2, -1)$ and radius $2\sqrt{2}$.

(*a*) (i) Show that the radius of circle P is $4\sqrt{2}$.

(ii) Hence show that circles P and Q touch. **4**

(*b*) Find the equation of the tangent to the circle Q at the point $(-4, 1)$. **3**

(*c*) The tangent in (*b*) intersects circle P in two points. Find the x-coordinates of the points of intersection, expressing you answers in the form $a \pm b\sqrt{3}$. **3**

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
(*a*)	2	C	CN	G9	proof	2001 P1 Q11
(*a*)	2	A/B	CN	G14		
(*b*)	3	C	CN	G11	$y = x + 5$	
(*c*)	3	C	CN	G12	$x = 2 \pm 2\sqrt{3}$	

•1 ic: interpret centre of circle (P)	•1 $C_P = (4, 5)$
•2 ss: find radius of circle (P)	•2 $r_P = \sqrt{16 + 25 - 9} = \sqrt{32} = 4\sqrt{2}$
•3 ss: find sum of radii	•3 $r_P + r_Q = 4\sqrt{2} + 2\sqrt{2} = 6\sqrt{2}$
•4 pd: compare with distance between centres	•4 $C_PC_Q = \sqrt{6^2 + 6^2} = 6\sqrt{2}$ **and** "so touch"
•5 ss: find gradient of radius	•5 $m_r = -1$
•6 ss: use $m_1 m_2 = -1$	•6 $m_{tgt} = +1$
•7 ic: state equation of tangent	•7 $y - 1 = 1(x + 4)$
•8 ss: substitute linear into circle	•8 $x^2 + (x + 5)^2 - 8x - 10(x + 5) + 9 = 0$
•9 pd: express in standard form	•9 $2x^2 - 8x - 16 = 0$
•10 pd: solve (quadratic) equation	•10 $x = 2 \pm 2\sqrt{3}$

[SQA] 4. The point $P(2, 3)$ lies on the circle $(x + 1)^2 + (y - 1)^2 = 13$. Find the equation of the tangent at P. **4**

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
	4	C	CN	G11	$2y + 3x = 12$	2002 P1 Q1

•1 ic: interpret centre from equ. of circle	•1 $C = (-1, 1)$
•2 ss: know to find gradient of radius	•2 $m_{rad} = \frac{2}{3}$
•3 ss: know to find perp. gradient	•3 $m_{tgt} = -\frac{3}{2}$
•4 ic: state equation of tangent	•4 $y - 3 = -\frac{3}{2}(x - 2)$

[SQA] 5. For what range of values of k does the equation $x^2 + y^2 + 4kx - 2ky - k - 2 = 0$ represent a circle? **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
	5	A	NC	G9, A17	for all k	2000 P1 Q6

$\bullet^1$ ss: know to examine radius $\bullet^2$ pd: process $\bullet^3$ pd: process $\bullet^4$ ic: interpret quadratic inequation $\bullet^5$ ic: interpret quadratic inequation	$\bullet^1$ $g = 2k,\ f = -k,\ c = -k - 2$ *stated or implied by* $\bullet^2$ $\bullet^2$ $r^2 = 5k^2 + k + 2$ $\bullet^3$ (real $r \Rightarrow$) $5k^2 + k + 2 > 0$ (*accept* $\geq$) $\bullet^4$ use discr. **or** complete sq. **or** diff. $\bullet^5$ true for all k

[END OF QUESTIONS]