Mr Miscandlons Notes

National 5

Summary notes for National 5 mathematics.

% PERCENTAGES %

BASICS

"12% of" → $0.12 \times ?$

"12% increase" → $1.12 \times ?$

"12% decrease" → $0.88 \times ?$

$1.0 - 0.12$

COMPOUND INTEREST:

interest added every year.

LONG WAY:

1. calculate %
2. add it on
3. calculate %
4. add it on

SHORT WAY:

$\text{new} = \text{initial} \times (\text{multiplier})^{\text{years}}$

APPRECIATION ↑ (increase in value)
DEPRECIATION ↓ (decrease in value)

same method as compound interest (short way) unless percentage changes every year!

REVERSING THE CHANGE

1. find the new value as %
2. find 1%
3. ×100 to find original value

Ex A laptop costs £360 after being reduced by 10% in the sale. How much was it previously?

$90\% = £360$

$100\% - 10\%$

$1\% = 360 \div 90$

$= 40$

$100\% = 40 \times 100$

$= £400$

Rounding

You must be able to round to a certain number of decimal places or significant figures.

DECIMAL PLACES = number of digits after decimal point.
SIGNIFICANT FIGS = number of "non zero" digits

4632.6175

to 1 dp : 4632.6175
= 4632.6

to 2 dp : 4632.6175
= 4632.62

to 3 dp : 4632.6175
= 4632.618

to 1 sf : 4632.6175
= 5000 (not just 5)

to 2 sf : 4632.6175
= 4600

to 3 sf : 4632.6175
= 4630

SCALE FACTOR

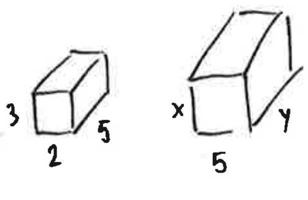
used with SIMILAR shapes

Scale factor > 1 for enlargement
scale factor < 1 for reduction

$$sf = \frac{\text{new}}{\text{old}}$$

LINEAR (single dimension)	AREA (two dimensions)	VOLUME (three dimensions)
$\frac{\text{new}}{\text{old}}$	$(l.s.f)^2$	$(l.s.f)^3$

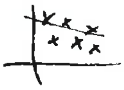
Ex.



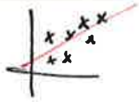
$l.s.f = \frac{\text{new}}{\text{old}} = \frac{5}{2}$

$a.s.f = \left(\frac{5}{2}\right)^2 = \frac{25}{4}$

$v.s.f = \left(\frac{5}{2}\right)^3 = \frac{125}{8}$

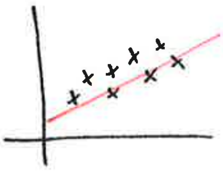


Scatter Graphs



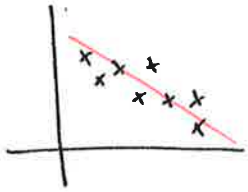
used to plot two sets of related data on one graph

CORRELATION:
(relationship between datasets)



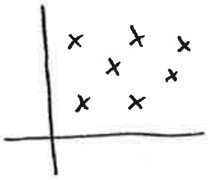
POSITIVE

As one increases the other increases



NEGATIVE

As one increases the other decreases



NONE

No pattern!

LINE OF BEST FIT:

this is a line that best represents the correlation - in theory there should be a similar number of crosses above and below the line.

You'll have to use your knowledge of straight line $y = mx + c$

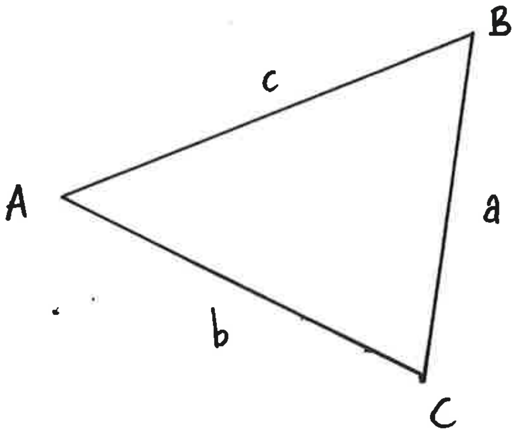
REMEMBER: You must write the equation based on info you have. If the axes are H & t then $H = mt + c$

Can you? Use the equation of the line of best fit to estimate a value given the other?

NON RIGHT ANGLED TRIG

You may be asked to do this non-calc if given $\sin A = ?$ or $\cos C = ?$

CAN YOU DO THIS IF ASKED?

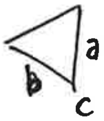


UPPER CASE LETTERS = VERTICES

lower case letters = sides

AREA:

$$\text{Area} = \frac{1}{2} ab \sin C$$



NEED: two sides and inclusive angle

You must rewrite based on the letters given eg $A = \frac{1}{2} pq \sin R$

SINE RULE:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

(side ÷ sin of opposite angle)

NEED: Pairs of angle / opp side

Again, rewrite based on letters given.

SAME EQUATION CAN BE USE FOR FINDING EITHER A SIDE OR ANGLE

COSINE RULE:

to find a side ....

$$a^2 = b^2 + c^2 - 2bc \cos A$$

a | A

just remember to square root !!

NEED: two sides and inclusive angle

to find an angle ....

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

A / a

just remember to "shift cos"

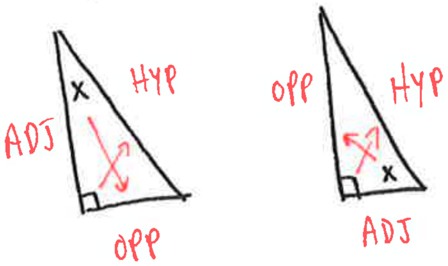
NEED: all three sides

NOT SURE WHAT RULE? TRY STARTING WITH COSINE RULE!

Right angled triG

(SOHCAHTOA)

LABEL THE TRIANGLE:



Hyp - hypotenus : always opposite the right angle

opp - opposite : opposite the (un)known angle

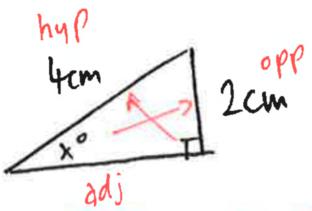
ADJ - adjacent : next to the (un)known angle

SOH CAH TOA

(sin) = $\frac{\text{opposite}}{\text{hypotenuse}}$ (cos) = $\frac{\text{adjacent}}{\text{hypotenuse}}$ (tan) = $\frac{\text{opposite}}{\text{adjacent}}$

FINDING A SIDE OR ANGLE:

1. LABEL TRIANGLE
2. USE SOHCAHTOA TO IDENTIFY TRIG RATIO
3. SOLVE (IF FINDING AN ANGLE USE SHIFT)



SOH CAH TOA

$\sin x = \frac{opp}{hyp}$

trig identities

trickier stuff

You must memorise these and how to use them!

$$\sin^2 x + \cos^2 x = 1$$

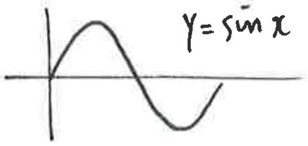
$$\tan x = \frac{\sin x}{\cos x}$$

Trigonometry

Graphs

Sin x

$y = \sin x$



max @ 90° = 1

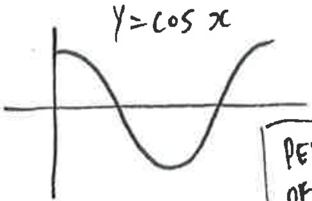
min @ 270° = −1

cuts x-axis @ 0°, 180°, 360°

$\boxed{\text{PERIOD OF } 360°}$

Cos x

$y = \cos x$



$\boxed{\text{PERIOD OF } 360°}$

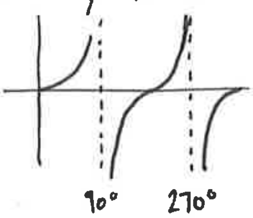
max @ 0° & 360° = 1

min @ 180° = −1

cuts x-axis @ 90°, 270°

Tan x

$y = \tan x$



$\boxed{\text{PERIOD OF } 180°}$

undefined @ 90°, 270°

cuts x-axis @ 0°, 180°, 360°

Remember:

$y = a \sin bx + c$

$y = a \cos bx + c$

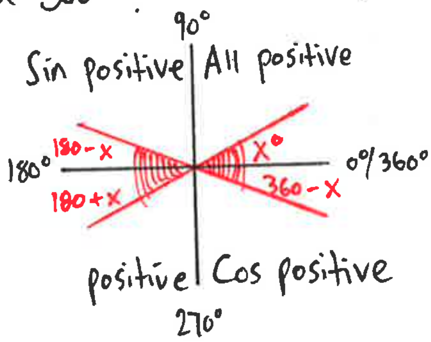
a = amplitude ↕ stretch/compress

b = period ↔

c = shift on y-axis ↕ move

Equations

Usually looking for <u>two</u> solutions between 0° and 360°.



Steps:

1. rearrange
2. solve for acute angle
3. find relevant quadrants
4. state solutions

$$2\sin x - 1 = 0$$

$$\sin x = \frac{1}{2}$$

✓S	A✓
T	C

$$x = 30°,\ 180° - 30°$$

$$= 30°,\ 150°$$

$$2\cos x + \sqrt{3} = 0$$

$$\cos x = -\frac{\sqrt{3}}{2}$$

+ve

$$aaa = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$= 30°$$

✓ −S	A+
✓ −T	C+

$$x = 180 - 30,\ 180 + 30$$

$$= 150°,\ 210°$$

STATISTICS

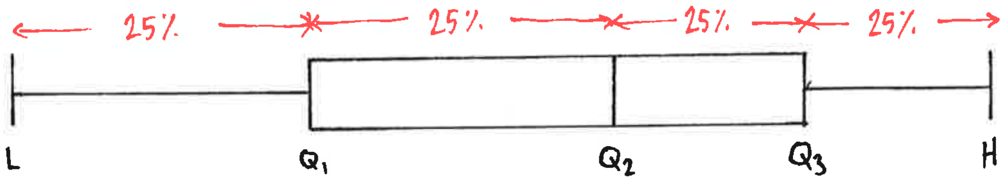
MEAN add up all the numbers then divide by number of numbers

MEDIAN middle number when numbers are in order

MODE most common

RANGE highest minus lowest

BOX PLOT:



L: lowest number in dataset

Q_1: "median" of bottom half

Q_2: median

Q_3: "median" of top half

H: highest number in dataset

Interquartile range (IQR) = $Q_3 - Q_1$

Semi Interquartile range (SIQR) = $\dfrac{Q_3 - Q_1}{2}$

measures spread of data!

STANDARD DEVIATION

$$SD = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}$$

this formula is given but just use the table !!!

1. Calculate $\bar{x}$ (mean)
2. Complete table

x	$x-\bar{x}$	$(x-\bar{x})^2$
values		ALL POSITIVE VALUES

$\boxed{\text{ADD UP}} = \Sigma(x-\bar{x})^2$

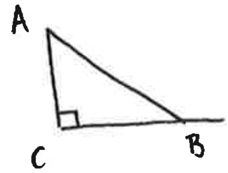
$$SD = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}$$

number of pieces of data.

3. Sub into equation

Pythagoras Theorem

ONLY occurs in right angled triangles. Used to find the length of one side of a triangle if the other two are known.



Finding the long side: (hypotenus)

$AB^2 = AC^2 + BC^2$

long side = add

Finding a shorter side:

$BC^2 = AB^2 - AC^2$

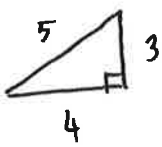
or $AC^2 = AB^2 - BC^2$

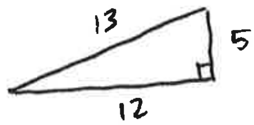
short side = subtract

REMEMBER: Always square root your answer!

PYTHAGOREAN TRIPLES:

These may occur in non-calc papers .... it would be good to be able to spot them.





$3^2 + 4^2 = 5^2$

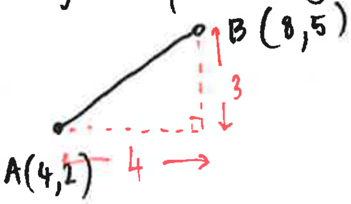
OR $6^2 + 8^2 = 10^2$

$5^2 + 12^2 = 13^2$

OR $10^2 + 24^2 = 26^2$

What is the shortest distance between two points?

- It's always a straight line, and we can use Pythagoras to calculate its length by creating a triangle.



$AB^2 = 4^2 + 3^2$

$= 25$

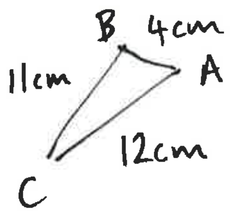
$AB = \sqrt{25}$

$= 5$

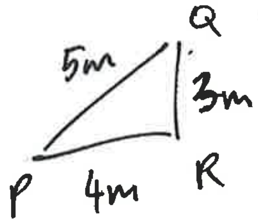
Converse of Pythagoras theorem ☆

This is used to prove if a triangle is or is not right angled.

Is triangle ABC or PQR right angled?



← Please draw better triangles than these →



SHORTER SIDES	LONGER SIDE
$AB^2 + BC^2$	AC^2
$= 4^2 + 11^2$	$= 12^2$
$= 16 + 121$	$= 144$
$= 137$	

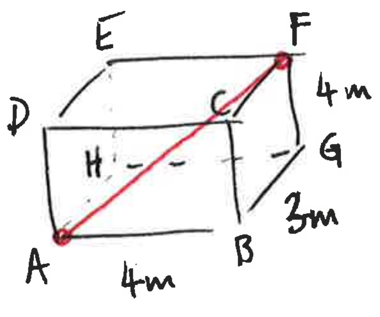
$AB^2 + BC^2 \neq AC^2$ so ABC is not a right angled triangle

SHORTER SIDES	LONGER SIDE
$PR^2 + QR^2$	PQ^2
$= 4^2 + 3^2$	$= 5^2$
$= 16 + 9$	$= 25$
$= 25$	

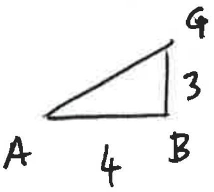
$PR^2 + QR^2 = PQ^2$ so PQR is a right angled triangle.

PYTHAGORAS IN 3D

Visualise the 3D shape, then Pythagoras (usually twice)



Calculate the length of AF.

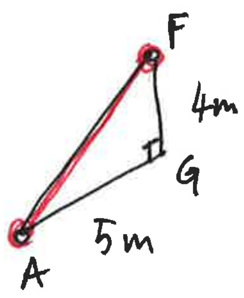


$AG^2 = AB^2 + BG^2$

$= 4^2 + 3^2$

$= 25$

$AG = 5m$



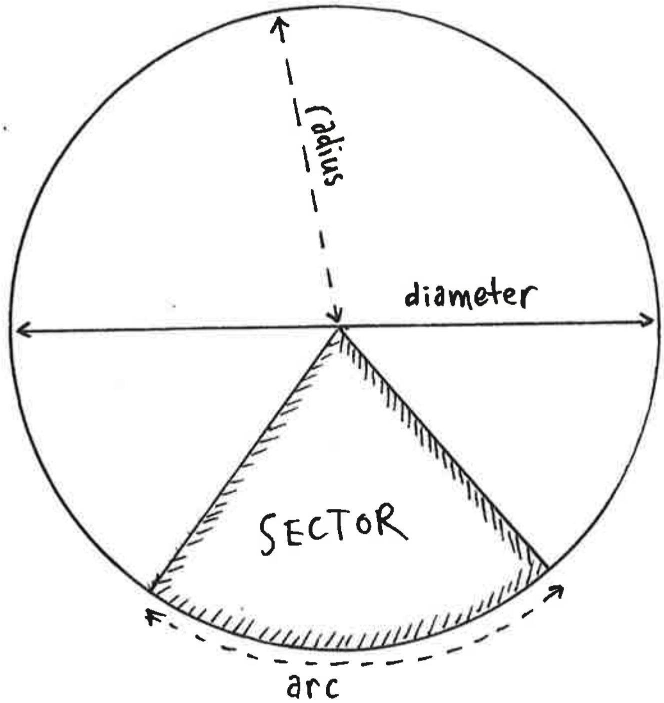
$AF^2 = AG^2 + FG^2$

$= 5^2 + 4^2$

$= 41$

$AF = 6.4m$

Circle (inc arcs & sectors)



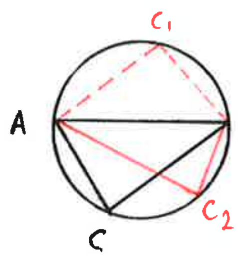
You will need to memorise every formula relating to the circle. These are:

CIRCUMFERENCE:	$C = \pi d$	AREA:	$A = \pi r^2$
LENGTH OF ARC:	$\text{Arc} = \frac{\text{angle}}{360°} \times \pi d$	AREA OF SECTOR:	$A = \frac{\text{angle}}{360°} \times \pi r^2$

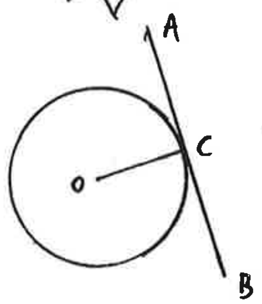
SECTORS:

ARCS:

ANGLES IN A CIRCLE:



If AB is a diameter then $\angle ACB$ is always 90°



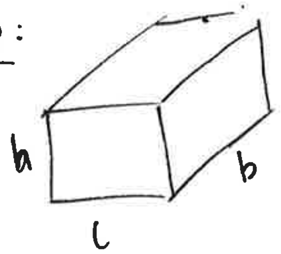
if AB is a tangent then $\angle ACO$ is always 90°.

VOLUME

We must know how to calculate the volume of ....

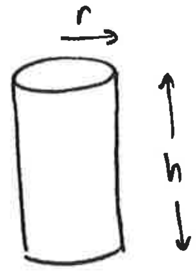
★ must memorise
• given in formulae list

★ **CUBOID**:



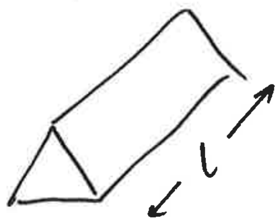
$V = lbh$

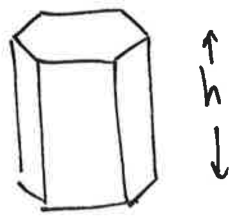
★ **CYLINDER**:



$V = \pi r^2 h$

★ **PRISM**:

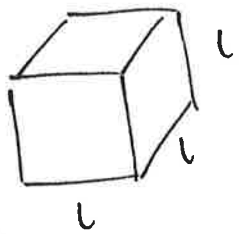




$V = A \times l$ OR $V = A \times h$

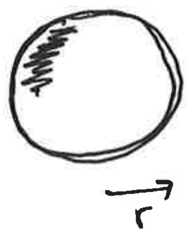
Area of cross section

★ **CUBE**:



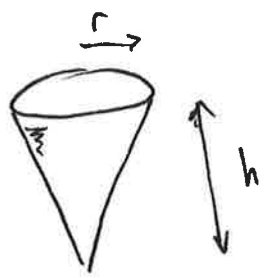
$V = l^3$

• **SPHERE**:



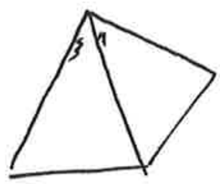
$V = \frac{4}{3}\pi r^3$

• **CONE**:



$V = \frac{1}{3}\pi r^2 h$

• **PYRAMID**:



V =

REMEMBER: ALWAYS STATE UNITS^3 !!

COMPOSITE SHAPES: This is when the 3D shape is made up with two or more known shapes.
Split into known shapes, calculate volumes & add together.

ALREADY KNOW THE VOLUME? You may need to calculate one dimension if given the volume ... fill in what you know & work backwards.

TRICKIER STUFF

Fractions

ADDING AND SUBTRACTING

1. COMMON DENOMINATOR
2. ADD / SUBTRACT NUMERATORS
3. KEEP DENOMINATOR
4. SIMPLIFY

$$\frac{2}{3} + \frac{1}{4} = \frac{8}{12} + \frac{3}{12} = \frac{11}{12}$$

$$3\frac{1}{2} - 1\frac{3}{4} = \frac{7}{2} - \frac{7}{4} = \frac{14}{4} - \frac{7}{4} = \frac{7}{4} = 1\frac{3}{4}$$

MULTIPLYING

1. TIMES NUMERATORS
2. TIMES DENOMINATORS
3. SIMPLYFY

$$\frac{3}{4} \times \frac{5}{6} = \frac{15}{24} = \frac{5}{8}$$

$$2\frac{1}{2} \times 3\frac{1}{3} = \frac{5}{2} \times \frac{10}{3} = \frac{50}{6} = 8\frac{2}{6} = 8\frac{1}{3}$$

DIVIDING

1. FLIP SECOND FRACTION
2. TIMES NUMERATORS
3. TIMES DENOMINATORS
4. SIMPLIFY

$$\frac{3}{4} \div \frac{5}{6} = \frac{3}{4} \times \frac{6}{5} = \frac{18}{20} = \frac{9}{10}$$

$$2\frac{1}{2} \div 3\frac{1}{3} = \frac{5}{2} \div \frac{10}{3} = \frac{5}{2} \times \frac{3}{10} = \frac{15}{20} = \frac{3}{4}$$

REMEMBER TO CONVERT TO IMPROPER FRACTIONS BEFORE WORKING

MIXED NUMBERS ⟷ IMPROPER FRACTIONS

$$4\frac{2}{3} = \frac{4 \times 3 + 2}{3} = \frac{14}{3}$$

$$\frac{14}{3} \rightarrow \begin{array}{r} 4 \text{ r } 2 \\ 3\overline{)14} \end{array} \rightarrow 4\frac{2}{3}$$

(expanding)(brackets)

single (bracket) : outside term times every term inside the bracket

$$3(x+4)$$
$$= 3 \times x + 3 \times 4$$
$$= 3x + 12$$

$$a(5-a)$$
$$= 5 \times a - a \times a$$
$$= 5a - a^2$$

$$3p(2q + 4p)$$
$$= 3p \times 2q + 3p \times 4p$$
$$= 6pq + 12p^2$$

(double)(brackets) : every term in first bracket times every term in second bracket.

$$(a+3)(a-3)$$
$$= a \times a + 3 \times a - 3 \times a + 3 \times (-3)$$
$$= a^2 + 3a - 3a - 9$$
$$= a^2 - 9$$

$$(q+2)(3q+1)$$
$$= q \times 3q + 2 \times 3q + 1 \times q + 2 \times 1$$
$$= 3q^2 + 6q + q + 2$$
$$= 3q^2 + 7q + 2$$

$$(x+3)^2$$
$$= (x+3)(x+3)$$
$$= x^2 + 3x + 3x + 9$$
$$= x^2 + 6x + 9$$

$$(x+3)(x^2 + 3x - 2)$$
$$= x^3 + 3x^2 - 2x + 3x^2 + 9x - 6$$
$$= x^3 + 6x^2 + 7x - 6$$

TRICKY ONES:

$$2(m+3) - (50-m)^2$$
$$= 2m + 6 - (50-m)(50-m)$$
$$= 2m + 6 - (2500 - 100m + m^2)$$
$$= -m^2 + 102m - 2496$$

REMEMBER TO COLLECT LIKE TERMS:

$$2(3x+2) - 3(2x-4)$$
$$= 6x + 4 - 6x + 12$$
$$= 16$$

$$3x - 4x(x-2)$$
$$= 3x - 4x^2 + 8x$$
$$= 11x - 4x^2$$

Factorising

Putting stuff in brackets

① Common factor - remove the highest common factor

$$4x^2 - 12 = 4(x^2 - 3)$$

$$7xy + 14x^2 = 7x(y + 2x)$$

② Difference of two Squares (DOTS) - both terms are squared and there is a minus sign.

$$x^2 - y^2 = (x - y)(x + y)$$

$$4 - 9x^2 = (2 - 3x)(2 + 3x)$$

$$x^4 - 25 = (x^2 - 5)(x^2 + 5)$$

③ Trinomials - three terms $[ax^2 + bx + c]$

multiply to give c

add to give b.

$$x^2 - 2x - 8 = (x - 4)(x + 2)$$

-8		
-2	4	(2)
2	-4	(-2) ✓
8	-1	(7)
-8	1	(-7)

$$x^2 + 8x + 15 = (x + 3)(x + 5)$$

15		
3	5	(8) ✓
1	15	(16)

GENERAL RULE :

$$x^2 + bx + c = (x + \quad)(x + \quad)$$

$$x^2 + bx - c = (x + \quad)(x - \quad)$$

$$x^2 - bx + c = (x - \quad)(x - \quad)$$

$$x^2 - bx - c = (x + \quad)(x - \quad)$$

4. Trinomials with a number infront of x^2

This requires a table!

$$2x^2 + 7x + 3$$

factors $\to$ ($2x^2$)   factors (3)

$$\begin{array}{c} 2x^2 \\ \hline 2x \quad x \end{array}$$

$$\begin{array}{cc} \multicolumn{2}{c}{3} \\ \hline 3 & 1 \\ 1 & 3 \\ -3 & -1 \\ -1 & -3 \end{array}$$

You must "swap" positions of the factors when there is a number infront of x^2

TABLE LOOKS LIKE :

$$\begin{array}{c|c} 2x & 3 \\ \hline x & 1 \end{array} \qquad \left.\begin{array}{c} 3x \\ 2x \end{array}\right\} 5x \quad ✗$$

$$\begin{array}{c|c} 2x & 1 \\ \hline x & 3 \end{array} \qquad \left.\begin{array}{c} x \\ 6x \end{array}\right\} 7x \quad ✓$$

So ...

$$2x^2 + 7x + 3$$
$$= (2x+1)(x+3)$$

top line of table   bottom line of table

5. Order of factorising

We <u>must</u> follow an order when factorising

1. Common factor
2. DOTS
3. Trinomials

<u>Ex</u>

$$5x^2 - 125$$
$$= 5(x^2 - 25)$$
$$= 5(x-5)(x+5)$$

$$3x^2 - 6x - 24$$
$$= 3(x^2 - 2x - 8)$$
$$= 3(x-4)(x+2)$$

Equations and Inequations

Aren't they the same?

When solving equations whatever we do to one side we do to the other.

$$3x + 4 = x + 14$$
$$-x \qquad -x$$
$$2x + 4 = 14$$
$$-4 \qquad -4$$
$$2x = 10$$
$$\div 2 \qquad \div 2$$
$$x = 5$$

$$3x + 4 = x + 14$$
$$3x - x = 14 - 4$$
$$2x = 10$$
$$x = 5$$

When fractions are involved, multiply to get rid of it.

$$\frac{1}{2}(3x + 2) = 28$$
$$\times 2 \qquad \times 2$$
$$3x + 2 = 56$$
$$-2 \qquad -2$$
$$3x = 54$$
$$\div 3 \qquad \div 3$$
$$x = 18$$

$$3y + 6 = 8 + 9y$$
$$-9y \qquad -9y$$
$$-6y + 6 = 8$$
$$-6 \qquad -6$$
$$-6y = 2$$
$$\div(-6) \qquad \div(-6)$$
$$y = \frac{2}{-6}$$

$$y = -\frac{1}{3}$$ ← you can have a fraction as an answer!

Remember: treat inequations like equations, it has a different symbol but the working is the same.

BE CAREFUL: WHEN DIVIDING BY A NEGATIVE THE INEQUALITY CHANGES DIRECTION.

$$3p + 2 \geq 5p + 6$$

$-5p \qquad -5p$

$$-2p + 2 \geq 6$$

$-2 \qquad -2$

$$-2p \geq 4$$

$\div -2 \qquad \div -2$

$$p \leq -2$$

$$3(r + 2) < r - 12$$

$$3r + 6 < r - 12$$

$-r \qquad -r$

$$2r + 6 < -12$$

$-6 \qquad -6$

$$2r < -18$$

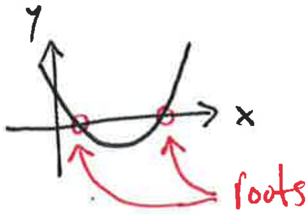
$\div 2 \qquad \div 2$

$$r < -9$$

sign stays the same!

Quadratics

How to solve equations (finding the roots)



- graphically – where the graph cuts the x-axis
- factorising – equate to zero, factorise, solve
- quadratic formula – $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Solve : $x^2 + 3x + 2 = 0$

$(x+2)(x+1) = 0$

$x + 2 = 0$ or $x + 1 = 0$

$x = -2$ $\quad$ $x = -1$

So graph cuts x-axis at $x = -1$ and $x = -2$

OR

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\quad$ $a = 1$ $\quad b = 3$ $\quad c = 2$

$x = \dfrac{-3 \pm \sqrt{(3)^2 - (4 \times 1 \times 2)}}{2}$

$x = \dfrac{-3 \pm \sqrt{1}}{2}$

$x = \dfrac{-3+1}{2}$ or $x = \dfrac{-3-1}{2}$

$= -1$ $\quad$ $= -2$

MORE TO THINK ABOUT :

$t^2 - 49 = 0$

$(t-7)(t+7) = 0$

$t = 7$ or $t = -7$

$4x^2 - 2x = 0$

$2x(2x-1) = 0$

$2x = 0$ $\quad$ $2x - 1 = 0$

$x = 0$ $\quad$ $2x = 1$

$x = \frac{1}{2}$

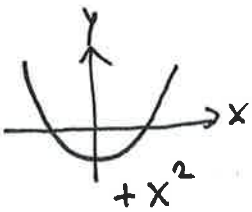
To describe the roots we use the discriminant :

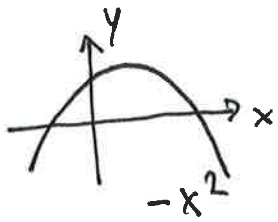
$b^2 - 4ac > 0$ two distinct roots (cuts axis twice)

$b^2 - 4ac = 0$ one root (touches axis once)

$b^2 - 4ac < 0$ no roots (doesn't touch axis)

PARABOLAS :





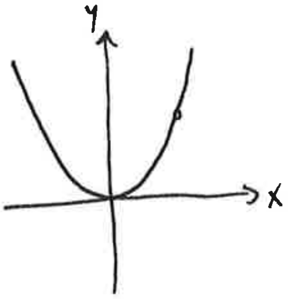
--- MORE QUADRATICS ---

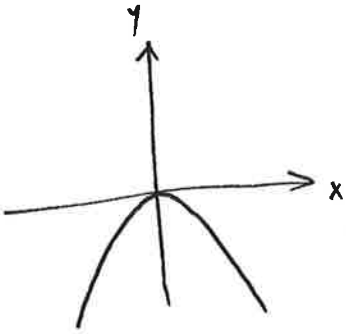
GRAPHS:

(a, b)

(a,b)

$y = (x-a)^2 + b$. $y = -(x-a)^2 + b$





$y = kx^2$ $y = -kx^2$

Sub in any point on the parabola <u>except</u> (0,0)

You need to be able to write the equation of a parabola if given the graph.

SKETCHING $y = (x-a)(x-b)$:

Follow these rules:

1. Find roots (cuts x-axis) when $y = 0$.
2. Find y-int (cuts y-axis) when $x = 0$
3. Find turning point
4. Shape $+x^2$ $-x^2$
   ∪ ∩
5. Annotate!

You <u>must</u> show all key points on the sketch!

COMPLETE THE SQUARE:

Rewrite $y = x^2 + bx + c$ in the form
$y = \left(x + \frac{b}{2}\right)^2 + d$

Follow these rules:

1. Rearrange to $x^2 + bx + c$
2. Half value of b.
3. Subtract b^2 at the end

$$x^2 + 4x + 2$$
$$= (x+2)^2 + 2 - 4 \quad 2^2$$
$$= (x+2)^2 - 2$$

$$x^2 + 10x - 3$$
$$= (x+5)^2 - 3 - 25 \quad 5^2$$
$$= (x+5)^2 - 28$$

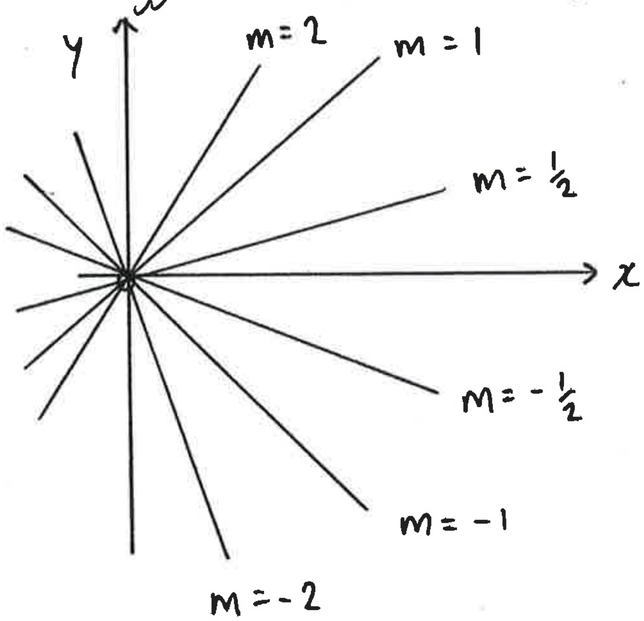
Straight line

Can you see the point?

Gradient = Steepness of a line = m

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad (x_1, y_1)(x_2, y_2)$$

If m is positive, line slopes upwards from left to right
If m is negative, line slopes downwards from left to right
Bigger value of m, steeper the line



EQUATION OF A LINE

$y = mx + c$

grad. ; y-int

$y - b = m(x - a)$

y coord of point (a, b) ; grad. ; x coord of point (a, b)

$Ax + By + C = 0$

rearranged to equal 0

<u>Parallel</u> lines have <u>same</u> gradient

Perpendicular lines, gradients multiply to give -1

$$m_1 \times m_2 = -1$$

To find the gradient of a given line <u>always</u> write as

$$y = mx + c.$$

SIMULTANEOUS EQUATIONS
SIMULTANEOUS EQUATIONS

We solve two equations with two unknowns by:

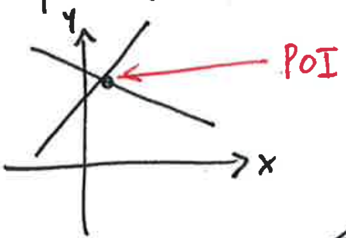
- graphically ← don't do it this way unless asked
- substitution
- elimination

graphically

- table of values

x	0	1	2
y	4	6	8

- plot both lines
- state point of intersection



substitution

- rearrange to $y = ?$
- sub $y = ?$ into other eqn
- solve for x
- sub x into equation to solve for y

Ex

$$x + 2y = 14$$
$$y = 2x + 2$$

$$x + 2(2x + 2) = 14$$
$$x + 4x + 4 = 14$$
$$5x = 10$$
$$x = 2$$

sub $x = 2$ into $y = 2x + 2$
$$= 2(2) + 2$$
$$= 6$$

elimination

- number infront of y must be the same in each equation (multiply if need be)
- same signs subtract
  different signs add
- solve for x, then for y

Ex

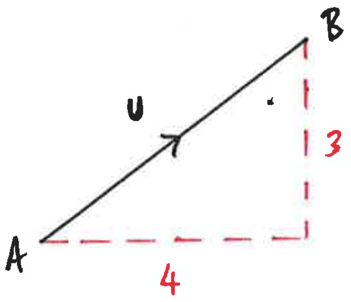
$$2x + y = 14 \quad \text{—(1)} \quad \times 2$$
$$3x + 2y = 18 \quad \text{—(2)}$$

$$4x + 2y = 28$$
$$3x + 2y = 18$$
$$- \quad x = 10$$
$$x = 10$$

Sub $x = 10$ into (1) $\quad 2(10) + y = 14$
$$20 + y = 14$$
$$y = -6$$

Vectors

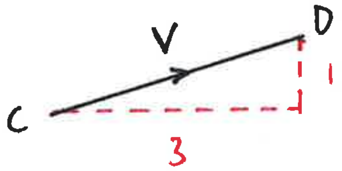
A vector has direction & magnitude!



$\vec{AB}$ in the diagram is a directed line segment

$\vec{AB} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ in component form

$u = \vec{AB}$



adding

$$\begin{aligned} & u + v \\ &= \vec{AB} + \vec{CD} \\ &= \begin{pmatrix} 4 \\ 3 \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} 7 \\ 4 \end{pmatrix} \end{aligned}$$

subtracting

$$\begin{aligned} & u - v \\ &= \vec{AB} - \vec{CD} \\ &= \begin{pmatrix} 4 \\ 3 \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 \\ 2 \end{pmatrix} \end{aligned}$$

multiplying

$$\begin{aligned} & 2u - 3v \\ &= 2\vec{AB} - 3\vec{CD} \\ &= 2\begin{pmatrix} 4 \\ 3 \end{pmatrix} - 3\begin{pmatrix} 3 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} 8 \\ 6 \end{pmatrix} - \begin{pmatrix} 9 \\ 3 \end{pmatrix} \\ &= \begin{pmatrix} -1 \\ 3 \end{pmatrix} \end{aligned}$$

Magnitude of a vector is the "length" or size

$$|u| = |\vec{AB}| = \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = 5 \text{ units}$$

$$|v| = |\vec{CD}| = \sqrt{3^2 + 1^2} = \sqrt{10}$$

SAME RULES FOR 3D VECTORS: $p = \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix}$

$$|p| = \sqrt{4^2 + (-2)^2 + 1^2} = \sqrt{16 + 4 + 1} = \sqrt{21}$$

Changing the subject

Changing the subject of a formula can be done several ways. Use the method you feel most confident with.

WHEN NEW SUBJECT ISN'T A DENOMINATOR:

$V = \pi r^2 h \quad [h]$ ← NEW SUBJECT

$\pi r^2 h = V$ ← FLIP SIDES

$h = \dfrac{V}{\pi r^2}$ ← DIVIDE BY πr^2

OR

$h \xrightarrow{\times \pi r^2} V$

$h \xleftarrow{\div \pi r^2} V$

$m = 4 - \sqrt{3u} \quad [u]$

$4 - \sqrt{3u} = m$ ← FLIP SIDES

$-\sqrt{3u} = m - 4$ ← SUBTRACT 4

$\sqrt{3u} = 4 - m$ ← $\times(-1)$

$3u = (4-m)^2$ ← SQUARE

$u = \dfrac{(4-m)^2}{3}$ ← DIVIDE BY 3

WHEN NEW SUBJECT IS A DENOMINATOR

$Q = \dfrac{4}{m^2 u} \quad [m]$

$m^2 u Q = 4$ ← $\times\, m^2 u$ TO HELP REARRANGE

$m^2 = \dfrac{4}{uQ}$ ← $\div\, uQ$

$m = \sqrt{\dfrac{4}{uQ}}$ ← SQUARE ROOT

$v = au + \dfrac{s}{t} \quad [t]$

$v - au = \dfrac{s}{t}$ ← ISOLATE FRACTION WITH t

$t(v - au) = s$ ← $\times\, t$ TO HELP REARRANGE

$t = \dfrac{s}{(v - au)}$ ← $\div\, (v - au)$

Surds

THE RULES: $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$ $\qquad$ $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Simplify:

$\sqrt{108}$
$= \sqrt{4} \times \sqrt{27}$
$= \sqrt{4} \times \sqrt{9} \times \sqrt{3}$
$= 2 \times 3 \times \sqrt{3}$
$= 6\sqrt{3}$

$\sqrt{96}$
$= \sqrt{4} \times \sqrt{24}$
$= \sqrt{4} \times \sqrt{4} \times \sqrt{6}$
$= 4\sqrt{6}$

$\sqrt{50} - \sqrt{8}$
$= \sqrt{25} \times \sqrt{2} - \sqrt{4} \times \sqrt{2}$
$= 5\sqrt{2} - 2\sqrt{2}$
$= 3\sqrt{2}$

Rationalising the denominator – getting rid of a root on the bottom of a fraction

$$\frac{a}{b\sqrt{c}} = \frac{a}{b\sqrt{c}} \times \frac{\sqrt{c}}{\sqrt{c}}$$

$\frac{1}{3\sqrt{2}}$
$= \frac{1}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$
$= \frac{\sqrt{2}}{6}$

$\frac{2}{5\sqrt{6}}$
$= \frac{2}{5\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$
$= \frac{2\sqrt{6}}{30}$
$= \frac{\sqrt{6}}{15}$

$\frac{\sqrt{5}}{2\sqrt{3}}$
$= \frac{\sqrt{5}}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$
$= \frac{\sqrt{15}}{6}$

indices

RULES :

MULTIPLYING

$$a^m \times a^n = a^{m+n}$$

$$a^3 \times a^4 = a^7$$

DIVIDING

$$a^m \div a^n = a^{m-n}$$

$$a^5 \div a^2 = a^3$$

POWERS OF POWERS

$$(a^m)^n = a^{mn}$$

$$(a^2)^5 = a^{10}$$

FRACTIONAL

$$\sqrt[n]{a^m} = a^{m/n}$$

$$\sqrt[4]{a^3} = a^{3/4}$$

NEGATIVE POWERS

$$a^{-n} = \frac{1}{a^n}$$

$$\frac{1}{a^2} = a^{-2} \qquad a^{-5} = \frac{1}{a^5}$$

a^0 AND a^1

$$a^1 = a \qquad 5^1 = 5$$

$$a^0 = 1 \qquad 5^0 = 1$$

You must know how to: write in index form
write to positive power
write variable as numerator
Simplify

EX

① $$\frac{3x^2 \times 5x^4}{20x^3}$$

$$= \frac{15x^{2+4}}{20x^3}$$

$$= \frac{15x^{6-3}}{20}$$

$$= \frac{3x^3}{4}$$

② $$\sqrt[3]{a^4} = a^{4/3}$$

③ $$16^{3/4} = \sqrt[4]{16}^3$$

$$= 2^3$$

$$= 8$$

④ $$\sqrt{x}\left(x^2 - \frac{1}{\sqrt{x}}\right)$$

$$= x^{1/2}(x^2 - x^{-1/2})$$

$$= x^{1/2+2} - x^{1/2-1/2}$$

$$= x^{5/2} - x^0$$

$$= x^{5/2} - 1$$

algebraic fractions

Same rules as standard fractions apply :

- $+$ common denominator needed
- $-$ common denominator needed
- $\times$ times numerators, times denominators, simplify
- $\div$ flip second fraction, change to multiply, multiply

SIMPLIFYING : Similar to standard fractions but usually involves factorising

eg.

$$\frac{5\cancel{a}}{7a^{\cancel{2}}} = \frac{5}{7a}$$

$$\frac{3b^2 + 12b}{5b^2 + 20b} = \frac{3b\cancel{(b+4)}}{5b\cancel{(b+4)}} = \frac{3\cancel{b}}{5\cancel{b}} = \frac{3}{5}$$

$$\frac{x^2 - 4}{x^2 + 3x + 2} = \frac{(x-2)\cancel{(x+2)}}{(x+1)\cancel{(x+2)}} = \frac{x-2}{x+1}$$

$$\frac{12x^2 - 3}{2x^2 + 5x - 3} = \frac{3(4x^2 - 1)}{(2x-1)(x+3)} = \frac{3\cancel{(2x-1)}(2x+1)}{\cancel{(2x-1)}(x+3)} = \frac{3(2x+1)}{x+3}$$

$$\frac{x^2 + 6x - 40}{x^2 - 100} = \frac{\cancel{(x+10)}(x-4)}{\cancel{(x+10)}(x-10)} = \frac{x-4}{x-10}$$