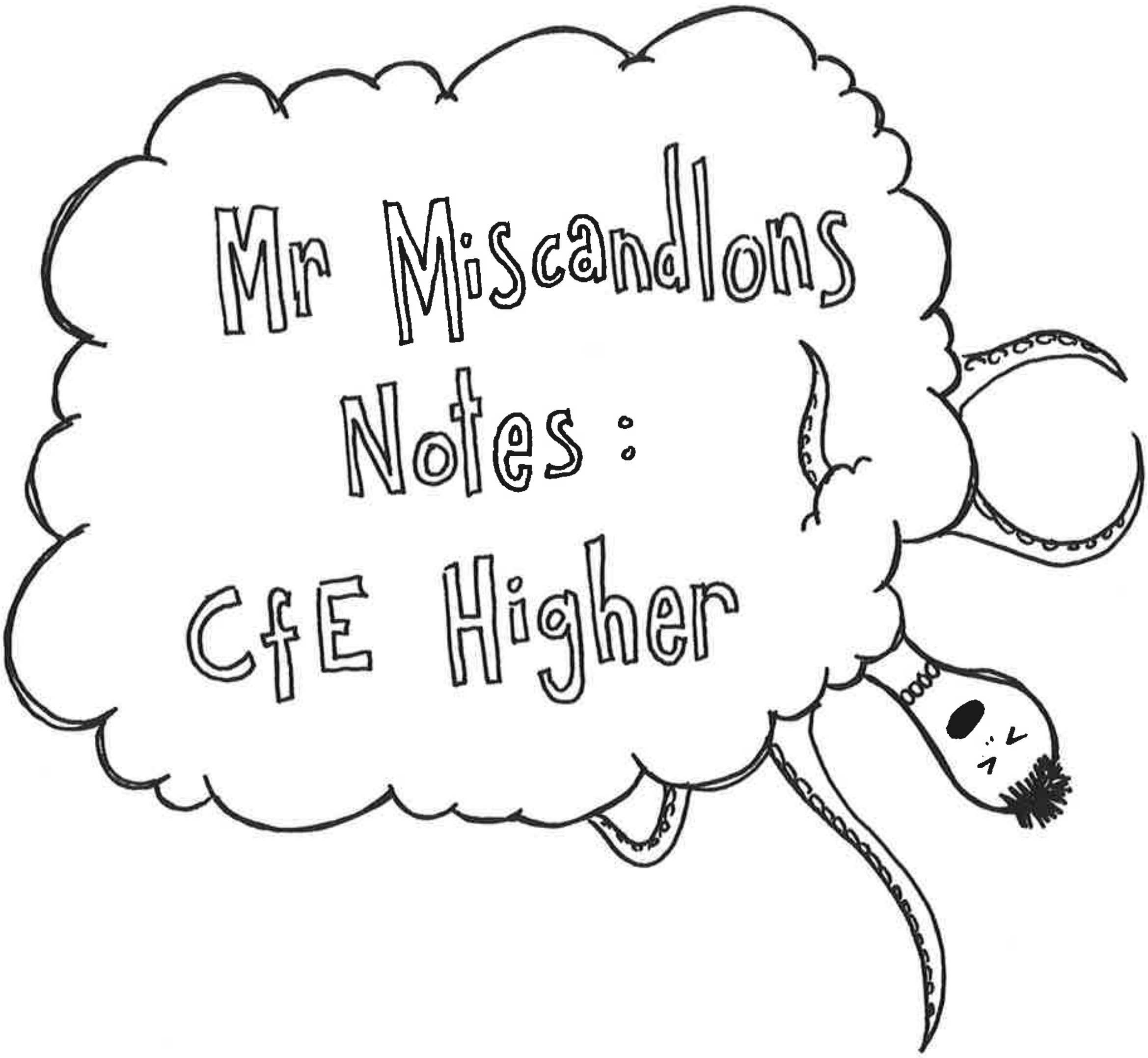

Mr Miscandlons
Notes :
CfE Higher

# Straight Line

## EQUATIONS :

$$y = mx + c$$

$$y - b = m(x - a)$$

$$Ax + By + C = 0$$

Where $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ (gradient)

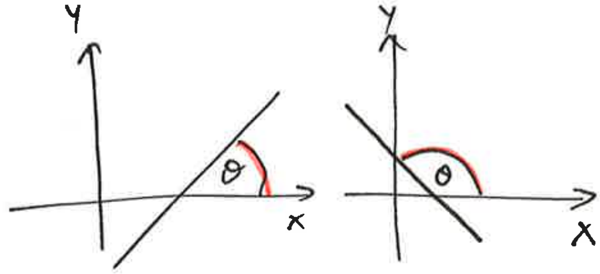
Point $(a,b)$ lies on the line

NOTE : NOT $(A,B)$!

## GRADIENT :

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ $(x_1, y_1)$ $(x_2, y_2)$ ← two coords on the line

$m = \tan\theta$




NOTE: θ ALWAYS ON RIGHT OF LINE

## PARALLEL & PERPENDICULAR LINES :

Parallel lines $m_1 = m_2$

Perpendicular lines $m_1 \times m_2 = -1$

★ TIP: TO FIND $m_\perp$ (M PERP) FLIP $m_1$ UPSIDE DOWN & CHANGE SIGN

$m_1 = \frac{2}{3}$ $m_\perp = -\frac{3}{2}$ ; $m_1 = 4$ $m_\perp = -\frac{1}{4}$

## POINTS OF INTERSECTION :

PoI is where two lines cross.

To find PoI :

1. Rearrange both equations to y=
2. Equate one to the other
3. Solve for $x$
4. Solve for $y$

## DOES A POINT LIE ON A LINE ?

METHOD 1 :

$y = mx + c$

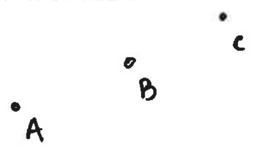
1. Sub in $x$ value from point
2. If $y$ is equal to the $y$ value of the point then it lies on line

METHOD 2 :

$Ax + By + C = 0$

1. Sub in $x$ and $y$ values from point
2. If equation equals zero then point lies on the line.

## COLLINEARITY (DO THREE POINTS LIE ON THE SAME STRAIGHT LINE?)




Collinear if $m_{AB} = m_{BC}$

Must state :

"$m_{AB} = m_{BC}$ so lines are parallel. Share a common point, B, so A, B & C are collinear"

## MIDPOINT OF A LINE :

$$\text{Midpoint} = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

REMEMBER THESE

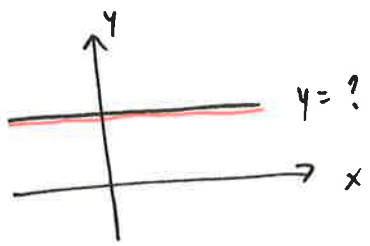
## LENGTH OF A LINE (DISTANCE BETWEEN TWO POINTS):

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

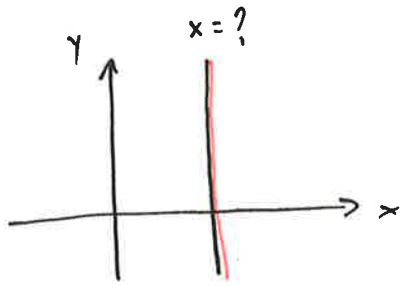
(FROM PYTHAGORAS THEOREM)

this is the distance formula !

## HORIZONTAL & VERTICAL LINES :

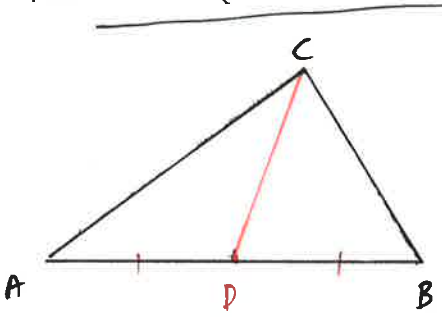



M = 0




m = undefined

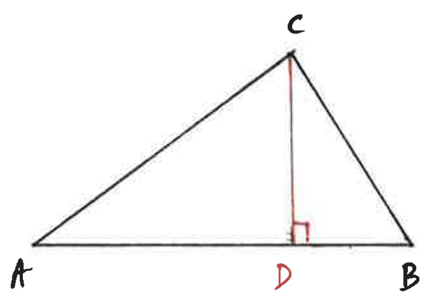
## TRIANGLES (MEDIAN, ALTITUDE & PERPENDICULAR BISECTOR):




Median from C or

Median of AB

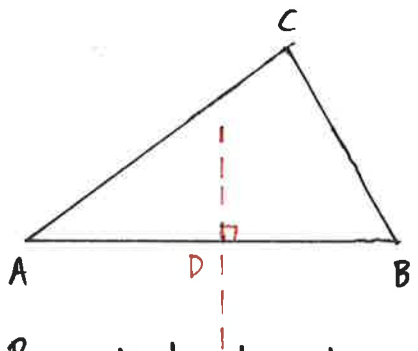
1. Find D using midpoint formula
2. Calculate $m_{CD}$ from C & D
3. Sub $m_{CD}$ & point C into $y - b = m(x - a)$




Altitude from C or

Altitude of AB

1. Find $M_{AB}$
2. Find $M_{CD}$ from $m_1 \times m_2 = -1$
3. Sub $m_{CD}$ & point C into $y - b = m(x - a)$




Perpendicular bisector of AB

1. Find D using midpoint formula
2. Find $M_{AB}$
3. Find $M_{\perp}$ from $m_1 \times m_2 = -1$
4. Sub in $m_{\perp}$ & point
into $y - b = m(x - a$

# functions & graphs

## COMPOSITE FUNCTIONS :

(substituting one function into another)

$f(x) = x^2 \qquad g(x) = x + 2$

$f(g(x)) = f(x+2)$
$= (x+2)^2$

$g(f(x)) = g(x^2)$
$= x^2 + 2$

$f(g(x)) \neq g(f(x))$

just work inside out !

NOTE : If $f(g(x)) = x$ then $f(x)$ & $g(x)$ are inverse functions

## INVERSE FUNCTIONS :

1. Set $f(x) = y$
2. rearrange to $x =$
3. substitute $x$ and $y$

$f(x) = 3x^2 + 2$

$y = 3x^2 + 2$

$3x^2 = y - 2$

$x^2 = \frac{y-2}{3}$

$x = \sqrt{\frac{y-2}{3}}$

$f^{-1}(x) = \sqrt{\frac{x-2}{3}}$

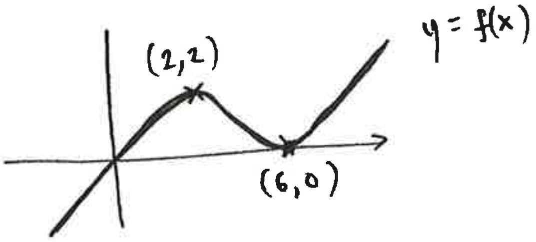
NOTE : Graphs of inverse functions are reflected in line $y = x$

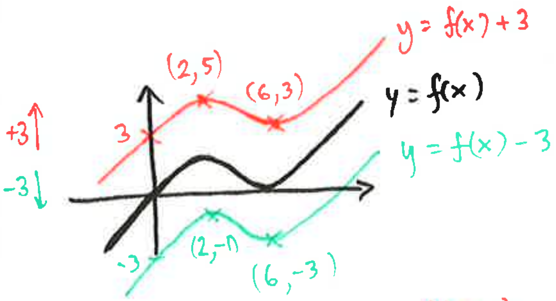
## RESTRICTED DOMAINS :

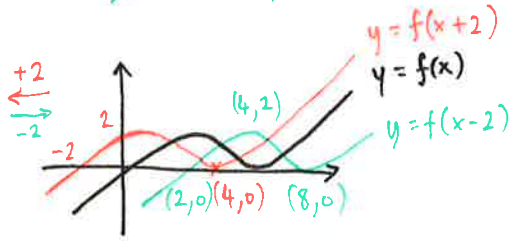
Denominator of fraction $\neq 0$

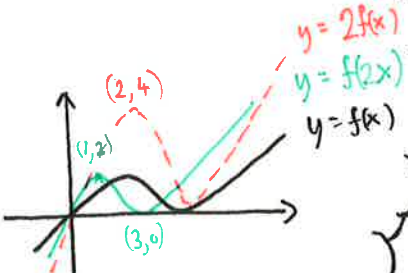
Number under root $\geq 0$

## TRANSFORMATION OF GRAPHS :




→ the miss hunter method states :

changes to x-coords :

$f(x+a)$ moves ↔ ( $+a$ to left, $-a$ to right )

$f(-x)$ flips in y axis

$f(kx)$ stretchs or compresses horizontally ( $k > 1$ comp... , $k < 1$ stretch... )

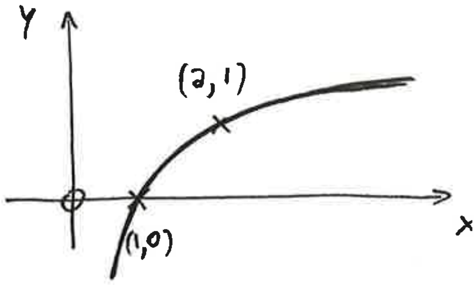
changes to y-coords :

$f(x) + a$ moves ↕ ( $+a$ up, $-a$ down )

$-f(x)$ flips in x axis

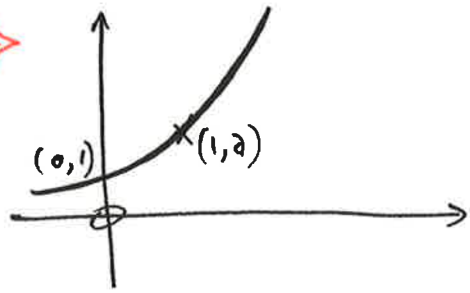
$kf(x)$ stretches or compresses vertically ( $k > 1$ stretch... , $k < 1$ comp... )

## SPECIAL GRAPHS :

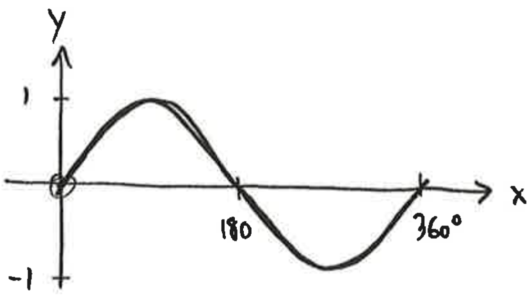



$y = \log_a x$

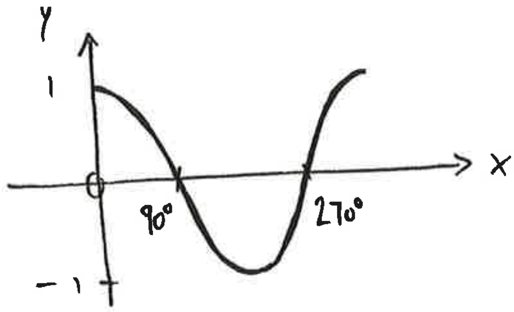
THESE ARE INVERSES OF EACH OTHER




$y = a^x$




$y = \sin x$




$y = \cos x$

You must be able to transform each of these !!

Trig graphs can also be written using radians

# Differentiation

**RULES:**

$f(x) = ax^n$

$f'(x) = nax^{n-1}$

"multiply by the power then decrease the power"

**REMEMBER:** Numbers differentiate to 0.

eg $y = 3x^4 + 5$

$\frac{dy}{dx} = 12x^3 + 0$

$= 12x^3$

$y \rightarrow \frac{dy}{dx}$

$f(x) \rightarrow f'(x)$

**INCREASING / DECREASING FUNCTIONS:**

$f'(x) > 0$ increasing

$f'(x) = 0$ stationary

$f'(x) < 0$ decreasing

**STATIONARY POINTS:**

Follow these steps:

1. Find $f'(x)$
2. Set $f'(x) = 0$ as SP $f'(x) = 0$
3. Solve $f'(x) = 0$ usually by factorising
4. Sub x values from 3 into y= for coordinates
5. Nature table
6. State points and nature

**PREPARING TO DIFFERENTIATE:**

We can't differentiate roots, brackets or $x$ as a denominator

$\sqrt[3]{x^2} \Rightarrow x^{2/3}$

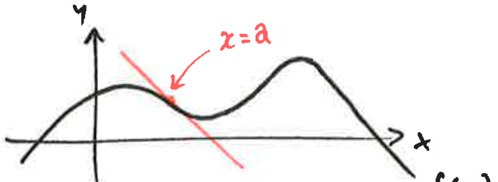
$x(x+2) \Rightarrow x^2 + 2x$

$\frac{3}{x^2} \Rightarrow 3x^{-2}$

THIS MUST BE DONE BEFORE DIFFERENTIATIN[G]

**GRADIENT OF TANGENT TO A CURVE:**

$f'(x) = m_{TAN}$ at $x$



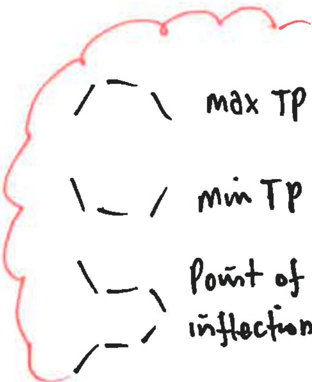

$m = f'(a)$

**EQUATION OF TANGENT TO A CURVE:**

Follow these steps:

1. Sub x value into f(x) to get coordinates of point of contact
2. Find $f'(x)$
3. Sub $x$ value into $f'(x)$ to find $m_{TAN}$
4. Sub in PoC & $m_{TAN}$ into $y - b = m(x - a)$

| x | $\rightarrow$ | a | $\rightarrow$ |
|---|---|---|---|
| f'(x) | "+ or −" | 0 | "+ or −" |
| Shape | / (+) | — | \ (−) |



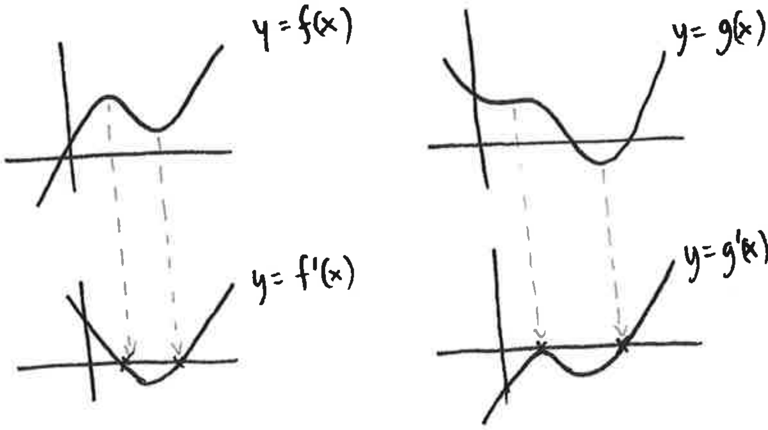
## GRAPH OF DERIVED FUNCTION :

Sketch $f(x)$ then $f'(x)$ on a seperate graph below

$f(x)$ SP $\rightarrow$ $f'(x)$ cuts x axis

$f(x)$ inc $\rightarrow$ $f'(x)$ above x-axis

$f(x)$ dec $\rightarrow$ $f'(x)$ below x-axis




## OPTIMISATION :

Finding a max or min value. First part of question is usually A/B level. Second part is just a stationary points question.

If you cant do part (a), leave it and come back to it or try working backwards.

## KEY WORDS :

Its likely to be differentiation when you see the phrases

"rate of change" "gradient of tangent"

How can you tell a function is never decreasing?

Find $\frac{dy}{dx}$ and prove it's never negative

(usually by completing the square)

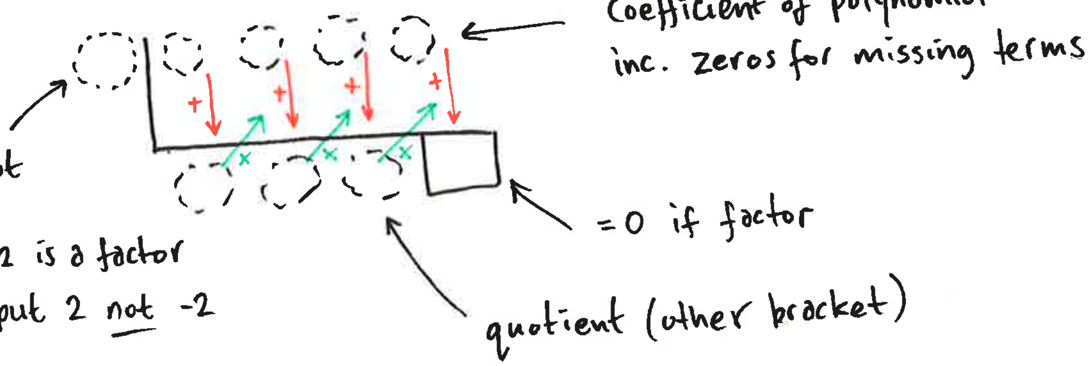
**POLYNOMIAL:** expression with degree $\geq 3$

# Polynomials and QUADRATICS

**QUADRATIC:** expression with degree $= 2$

## FACTORISING A POLYNOMIAL:

Use synthetic division




if $x-2$ is a factor you input 2 <u>not</u> $-2$

## FINDING AN UNKNOWN IN A POLYNOMIAL:

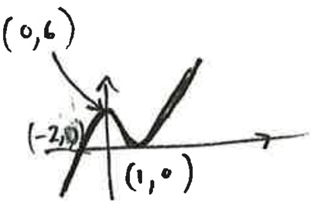
$3x^3 + 2x^2 + px + 5 = 0$. Find $p$ if $x=$ is a factor

Exact same process as factorising but set the remainder to equal zero and solve for unknown.

## GENERAL EQUATION OF A POLYNOMIAL:

$y = k(x-a)(x-b)(x-c)$ ← number brackets = number of roots

Use this to state equation of polynomial from graph.

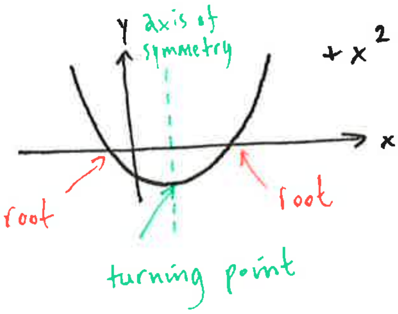


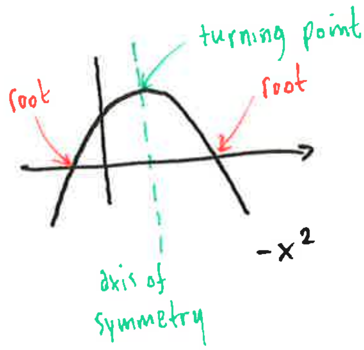

$y = k(x-a)(x-b)(x-c)$

$= k(x+2)(x-1)(x-1)$ – – – repeated root, graph only touches x-axis

Sub $y=6$ and $x=0$ into function to find $k$.

## QUADRATIC GRAPHS:




axis of symmetry is half way between roots

## NATURE OF ROOTS:

use the discriminant, if $ax^2 + bx + c = 0$

$b^2 - 4ac > 0$ two real roots (cuts x-axis twice)

$b^2 - 4ac = 0$ one real root (touches x-axis once)

$b^2 - 4ac < 0$ no real roots (doesnt cut x-axis)

in addition, if $b^2 - 4ac$ is a perfect square the roots are rational

## SOLVING A QUADRATIC:

If asked to solve, answer must include $x =$

EITHER

FACTORISE

$x^2 + bx + c = 0$

$(x + \quad)(x + \quad) = 0$

QUADRATIC FORMULA

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

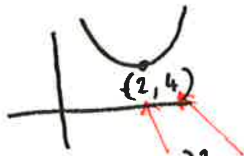
Memorise this

## COMPLETING THE SQUARE:

Rewrite $y = ax^2 + bx + c$ in the form

$y = a(x+p)^2 + q$

AoS $x = -p$ TP $= (-p, q)$




$y = 3(x-2)^2 + 4$

## TANGENT TO A PARABOLA:

(It's a touchy subject!)

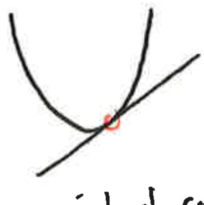
1. Set parabola to equal straight line
2. Rearrange to equal zero
3. Use discriminant

It's similar to nature of roots

$b^2 - 4ac > 0$

two PoI

$b^2 - 4ac = 0$



one point of contact (TANGENT)

$b^2 - 4ac < 0$

no PoI

# INTEGRATION

(OPPOSITE OF DIFFERENTIATION)

**RULES:**

$$\int ax^n \, dx = \frac{ax^{n+1}}{n+1} + c$$ ← dont forget the constant of integration

" increase the power then divide by the new power "

$$\int 5x^2 + 2x + 3 \, dx$$

$$= \frac{5x^3}{3} + \frac{2x^2}{2} + 3x + c$$

$$= \frac{5x^3}{3} + x^2 + 3x + C$$

tidy up!

## PREPARING TO INTEGRATE:

We can't integrate roots, brackets or x as a denominator

$$\sqrt[3]{x^2} \Rightarrow x^{2/3}$$

$$x(x+2) \Rightarrow x^2 + 2x$$

$$\frac{3}{x^2} \Rightarrow 3x^{-2}$$

THIS MUST BE DONE BEFORE INTEGRATING.

## INTEGRATING WITH LIMITS:

Answer will be a number not an expression!

$$\int_1^3 x \, dx$$ limits: 3 and 1

$$= \left[ \frac{x^2}{2} \right]_1^3$$ notice "c" doesn't appea

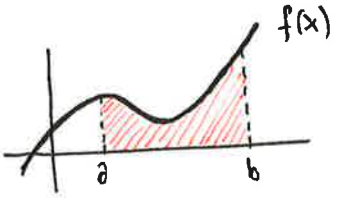
$$= \left( \frac{(3)^2}{2} \right) - \left( \frac{(1)^2}{2} \right)$$

evaluate when x=3 | evaluate when x=1

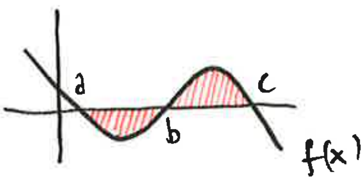
$$= \frac{9}{2} - \frac{1}{2}$$

$$= 4$$

## AREA UNDER A CURVE:




$$\int_a^b f(x) \, dx$$




$$\int_a^b f(x) \, dx$$ (ignore neg value)

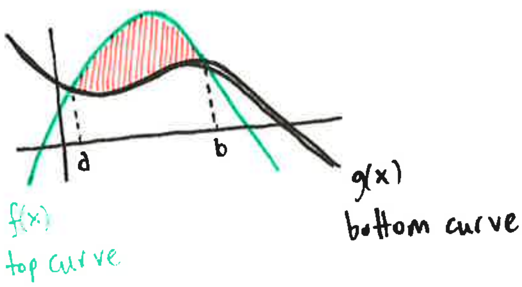
$$+$$

$$\int_b^c f(x) \, dx$$

REMEMBER UNITS : UNITS$^2$

## AREA BETWEEN TWO CURVES:

(OR A CURVE & STRAIGHT LINE)




$$\int_a^b f(x) - g(x) \, dx$$

$$\int_a^b \text{upper} - \text{lower} \, dx$$

## KEY WORDS:

" integrate " " area under | between "

" calculate f(x) if f'(x) = " " calculate y if $\frac{dy}{dx}$ = "

# all things trigonometric

## DEGREES & RADIANS:

$\pi$ radians $= 180°$

to convert

$$\text{degrees} \xrightarrow{\boxed{\times\pi\div180}} \text{radians}$$

$$\text{radians} \xrightarrow{\boxed{\times180\div\pi}} \text{degrees}$$

EXAMPLE

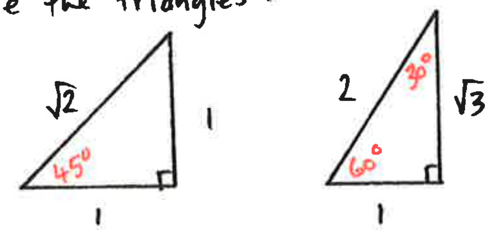
$$45° = \frac{45\pi}{180} = \frac{\pi}{4}$$

$$\frac{\pi}{4} = \frac{180\pi}{4\pi} = 45°$$

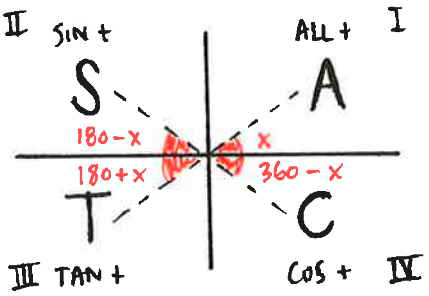
## EXACT VALUES:

| | $0°$ | $30°$ ($\frac{\pi}{6}$) | $45°$ ($\frac{\pi}{4}$) | $60°$ ($\frac{\pi}{3}$) | $90°$ ($\frac{\pi}{2}$) |
|---|---|---|---|---|---|
| $\sin x$ | 0 | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ | 1 |
| $\cos x$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ | 0 |
| $\tan x$ | 0 | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ | und. |

use the triangles:




## TRIG EQUATIONS & CAST DIAGRAM:




to solve

1. rearrange to trig function =
2. find first quadrant angle by ignoring sign
3. use CAST diagram & sign to identify quads
4. solve and state answers

$$\sqrt{2}\sin x + 1 = 0$$

$$\sqrt{2}\sin x = -1$$

$$\sin x = \frac{-1}{\sqrt{2}}$$

$$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$$

$\sin$ (−)


SOL III, IV $\quad x = 180 + 45,\ 360 - 45$

$= 225°,\ 315°$

## EQUATIONS WITH x AND 2x:

equations with mixed angles ($x°$ and $2x°$) cant be solved without replacing the $2x$ using:

GIVEN IN FORMULAE SHEET

$$\sin 2x = 2\sin x\cos x$$

$$\cos 2x = 2\cos^2 x - 1$$

$$= 1 - 2\sin^2 x$$

choose the right substitution then factorise

## COMPOUND ANGLE FORMULAE :

these are given in a formulae sheet, just know how to use them

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$ same sign

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$ opposite sign

## TRIG IDENTITIES :

Remember these ?

$$\sin^2 x + \cos^2 x = 1 \qquad \frac{\sin x}{\cos x} = \tan x$$

FOR TRIG GRAPHS SEE "FUNCTIONS & GRAPHS"

# Wave Function

allows use to add a cos and sin wave.
uses the compound angle formulae.

$$a\cos x + b\sin x = k\cos(x - \alpha)$$

Could be :
$k\cos(x \pm \alpha)$
$k\sin(x \pm \alpha)$

Follow this example :

$$4\sin x - 3\cos x = k\sin(x - \alpha)$$
$$= k\sin x\cos\alpha - k\cos x\sin\alpha$$

$-k\sin\alpha = -3$ so $k\sin\alpha = 3$

$k\cos\alpha = 4$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{3}{4}$$

| S ✓ | A ✓✓✓ |
|---|---|
| T ✓ | C ✓ |

$\alpha = 36.9°$

$$k = \sqrt{3^2 + 4^2}$$ ← from $k\sin\alpha$ and $k\cos\alpha$
$$= \sqrt{25}$$
$$= 5$$

finish with this statement → $$4\sin x - 3\cos x = 5\sin(x - 36.9)°$$

You could now use wave function to solve $4\sin x - 3\cos x = 1$ by rewriting to $5\sin(x - 36.9)° = 1$

# the Circle

## EQUATION OF A CIRCLE:

$$\boxed{(x-a)^2+(y-b)^2=r^2}$$

where centre $(a,b)$ radius $= r$

OR

$$\boxed{x^2+y^2+2gx+2fy+c=0}$$

where centre $(-g,-f)$ and

radius $= \sqrt{g^2+f^2-c}$

BOTH ARE GIVEN IN FORMULAE SHEET!

## POINTS OF INTERSECTION / TANGENCY:

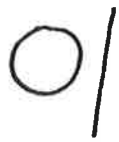
Substitute equation of line into the circle, use discriminant of resultant quadratic

$b^2-4ac>0$

two points

$b^2-4ac=0$

tangent



$b^2-4ac<0$

no points

similar to parabolas!

## DO TWO CIRCLES TOUCH?

to find out:
1. calculate $r_1$ and $r_2$
2. calculate distance between centres $(d)$
3. Compare

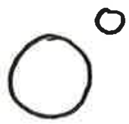
$r_1+r_2>d$

touch at two points

$r_1+r_2=d$

touch at one point

$r_1+r_2<d$



don't touch at all

★ two further cases occur when one circle is inside the other ★

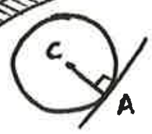
don't touch

$r_1-r_2>r$

touch at one point

$r_1-r_2=d$

CONCENTRIC: CIRCLES WITH THE SAME CENTRE




## EQUATION OF A TANGENT:

1. $M_{AC}$
2. $m_1 \times m_2 = -1$ for $M_2$
3. Use point A & $M_2$ for equation

# Recurrence Relations

$$U_{n+1} = aU_n + b$$

a sequence where each term is based on the previous term.

$a$ is usually a percentage in decimal form

## Does the sequence have a limit?

★ limit exists if $-1 < a < 1$ Remember to state this!

★ to find limit either: set $u_{n+1}$ and $u_n$ to $L$ and solve for $L$.

or $L = \dfrac{b}{1-a}$

★ You may be given three consecutive terms $u_1$, $u_2$ & $u_3$ and have to calculate $a$ & $b$ through simultaneous equations.

EXAMPLE: A scientist is studying a large flock of birds. Every minute 20% fly away but 20 new birds join. The initial size of the flock was 300 birds.

(a) write a recurrence relation to model the situation
(b) what can you say about the size of the flock in the long term?

(a) 20% fly away = 80% remain.

$$u_{n+1} = 0.8u_n + 20$$ ← number of "new" birds

(b) Size of flock will tend to a limit $-1 < 0.8 < 1$

$$L = 0.8L + 20$$

$$L - 0.8L = 20$$

$$0.2L = 20$$

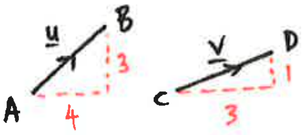
$$L = \frac{20}{0.2}$$ ← can you do this without a calculator?

$$L = 100$$

# Vectors

**RECAP FROM NAT 5:**

A vector has direction and magnitude




$\underline{u} = \overrightarrow{AB} = \begin{pmatrix}4\\3\end{pmatrix}$ $\quad \underline{v} = \overrightarrow{CD} = \begin{pmatrix}3\\1\end{pmatrix}$

$$2\underline{u} + 3\underline{v} = 2\begin{pmatrix}4\\3\end{pmatrix} + 3\begin{pmatrix}3\\1\end{pmatrix}$$
$$= \begin{pmatrix}8\\6\end{pmatrix} + \begin{pmatrix}9\\3\end{pmatrix}$$
$$= \begin{pmatrix}17\\9\end{pmatrix}$$

magnitude is length:

$$|\underline{u}| = \sqrt{4^2+3^2}$$
$$= \sqrt{25}$$
$$= 5 \text{ units}$$

RULES WORK WITH 3D VECTORS TOO!

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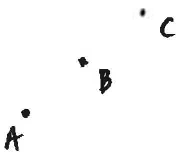
**COLLINEARITY:** Similar to straight line topic




1. Find $\overrightarrow{AB}$ & $\overrightarrow{BC}$
2. Check parallel
3. State "parallel and common point B so collinear"

**PARALLEL VECTORS:** parallel if one vector is a multiple of the other

**i, j, k:**

$\underline{i} = \begin{pmatrix}1\\0\\0\end{pmatrix}$ $\quad \underline{j} = \begin{pmatrix}0\\1\\0\end{pmatrix}$ $\quad \underline{k} = \begin{pmatrix}0\\0\\1\end{pmatrix}$

$\underline{i}$ is movement on x-axis

$\underline{j}$ is movement on y-axis

$\underline{k}$ is movement on z-axis

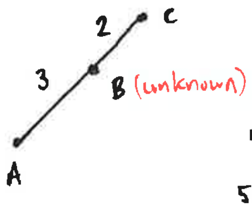
**SCALAR PRODUCT (DOT PRODUCT):** two ways to calculate, both are given in formulae sheet

$\underline{a}.\underline{b} = |\underline{a}||\underline{b}|\cos\theta$ ← angle between vectors $\underline{a}$ and $\underline{b}$

OR

$\underline{a}.\underline{b} = a_1b_1 + a_2b_2 + a_3b_3$

**FINDING COORDINATES OF A POINT ON A LINE:**




$\overrightarrow{AB} = \frac{3}{5}\overrightarrow{AC}$ ← 5 "parts" in whole line

$5\overrightarrow{AB} = 3\overrightarrow{AC}$

$5(\underline{b}-\underline{a}) = 3(\underline{c}-\underline{a})$

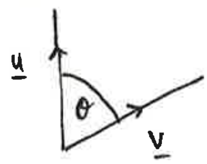
then solve!

REMEMBER: YOU MUST WRITE COORDINATES HORIZONTALLY!!!

☆ More of a challenge?

$\underline{a}.(\underline{b}+\underline{c}) = \underline{a}.\underline{b} + \underline{a}.\underline{c}$

**ANGLE BETWEEN VECTORS:**




$$\cos\theta = \frac{\underline{u}.\underline{v}}{|\underline{u}||\underline{v}|}$$

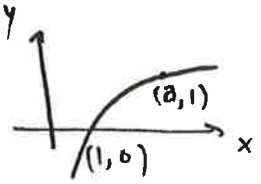
Not given in an exam, rearrange scalar product.

Vectors must be pointing away from each other!

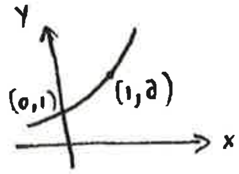
For perpendicular vectors: $\underline{a}.\underline{b} = 0$

# Logs & Exponentials

**GRAPHS:**




$y = \log_a x$




$y = a^x$

**QUESTION**

can you move these graphs or get back to the original graph if given a "moved" graph?

**WHAT IS A LOG?:**

$\log_3 9$ means "3 to the power of what equals 9?"

BASE

so $\log_3 9 = 2$

that means we can swap between logs & exponentials

$$\log_a y = x \iff y = a^x$$

BASE

**RULES OF LOGS:** these work both ways!

$$\log_a x + \log_a y = \log_a xy$$

$$\log_a x - \log_a y = \log_a\left(\frac{x}{y}\right)$$

MUST BE SAME BASE

$$\log_a x^n = n\log_a x$$

$$\log_a 1 = 0$$

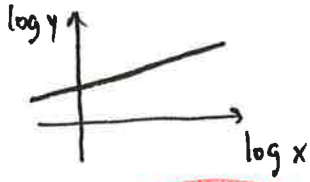
$$\log_a a = 1$$

You must remember these!

**EXPERIMENTAL DATA:**

this should always be a straight line

① Plot $\log y$ vs $\log x$




$y = kx^n$

$\log y = \log kx^n$

$= \log k + \log x^n$

$= \log k + n\log x$

$= n\log x + \log k$

so $\log k = c$ (y int)

$n$ = gradient

② Plot $\log y$ vs $x$


$y = ab^x$

$\log y = \log ab^x$

$= \log a + \log b^x$

$= \log a + x\log b$

$= x\log b + \log a$

So $\log a = c$ (y-int)

$\log b$ = gradient

# FURTHER CALCULUS

## DIFFERENTIATION

TRIG:

$y = \sin x$ $\quad \frac{dy}{dx} = \cos x$

$y = \cos x$ $\quad \frac{dy}{dx} = -\sin x$

GIVEN IN FORMULAE SHEET

REMEMBER: ALL ANGLES MUST BE IN RADIANS !!!

CHAIN RULE:

$$y = (ax+b)^n$$

$$\frac{dy}{dx} = n(ax+b)^{n-1} \times a$$

"multiply by power then decrease power by 1. also multiply by derivative of the bracket"

$$y = (4x+3)^5$$

$$\frac{dy}{dx} = 5(4x+3)^4 \times 4$$

$$= 20(4x+3)^4$$

$$y = (x^3+4)^7$$

$$\frac{dy}{dx} = 7(x^3+4)^6 \times 3x^2$$

$$= 21x^2(x^3+4)^6$$

## INTEGRATION

TRIG:

$$\int \sin x \, dx = -\cos x + c$$

$$\int \cos x \, dx = \sin x + c$$

$$\int \sin(ax+b)\, dx = -\frac{1}{a}\cos(ax+b) + c$$

$$\int \cos(ax+b)\, dx = \frac{1}{a}\sin(ax+b) + c$$

INTEGRATE $(ax+b)^n$:

$$\int (ax+b)^n \, dx$$

$$= \frac{(ax+b)^{n+1}}{a(n+1)} + c$$

"increase the power by 1 and divide by new power and derivative of bracket"

$$\int (4x+2)^5 \, dx$$

$$= \frac{(4x+2)^6}{6 \times 4} + c$$

$$= \frac{(4x+2)^6}{24} + c$$

$$\int (3x^2+1)^7 \, dx$$

$$= \frac{(3x^2+1)^8}{8 \times 6x} + c$$

$$= \frac{(3x^2+1)^8}{48x} + c$$